- Section 6.1 # 35

35. Suppose a vector **y** is orthogonal to vectors **u** and **v**. Show that **y** is orthogonal to the vector **u** + **v**.

Since y is orthogonal to u that means $y \cdot u = 0$.

Since y is orthogonal to v that means $y \cdot v = 0$.

Consequently

$$y \cdot (u+v) = y \cdot u + y \cdot v = 0 + 0 = 0$$

implies y is orthogonal to u+v,

- Section 6.4 # 23

23. Suppose $A = QR$, where Q is $m \times n$ and R is $n \times n$. Show that if the columns of A are linearly independent, then R must be invertible. [*Hint:* Study the equation $R\mathbf{x} = \mathbf{0}$ and use the fact that $A = QR$.]

Since R is square all we need to show is that there are no free variables. Thus we need show that $x=0$ is the only solution to $Rx=0$.

Suppose there was a solution $x \neq 0$ such that $Rx=0$. Then

$$Ax = QRx = Q0 = 0$$

Implies there is a non-zero vector such that $Ax=0$. This contradicts the assumption that the columns of A are independent. Therefore R must be invertible

- Section 6.4 # 24

24. Suppose $A = QR$, where R is an invertible matrix. Show that A and Q have the same column space. [*Hint:* Given $\mathbf{y}$ in Col A, show that $\mathbf{y} = Q\mathbf{x}$ for some $\mathbf{x}$. Also, given $\mathbf{y}$ in Col Q, show that $\mathbf{y} = A\mathbf{x}$ for some $\mathbf{x}$.]

To show $\text{Col } A = \text{Col } Q$ show containment in both directions.

Claim $\text{Col } A \subseteq \text{Col } Q$. Let $y \in \text{Col } A$. Then by definition of the column space $y = Az$ for some z.

Since $A = QR$ it follows that $y = QRz$. Taking $x = Rz$ then implies $y = Qx$ and so $y \in \text{Col } Q$.

Claim $\text{Col } Q \subseteq \text{Col } A$. Let $y \in \text{Col } Q$. Then by definition there is some w such that $y = Qw$. Let $x = R^{-1}w$. Then $Rx = w$ and substituting yields $y = QRx = Ax$.

Therefore $y \in \text{Col } A$.

Since $\text{Col } A \subseteq \text{Col } Q$ and $\text{Col } Q \subseteq \text{Col } A$ it follows $\text{Col } A = \text{Col } Q$.