Final exam is

Friday May 9

Friday, Second day of finals

10 a.m.	Monday/Wednesday/Friday (MWF)	10:15 a.m.-12:15 p.m.

in PE104

Systems of linear equations.

13. $1x_1 + 0x_2 - 3x_3 = 8$
$2x_1 + 2x_2 + 9x_3 = 7$
$0x_1 + 1x_2 + 5x_3 = -2$

A b

$$b = \begin{bmatrix} 8 \\ 7 \\ -2 \end{bmatrix}$$

Idea is to simplify the notation using matrices and vectors.
columns of matrix correspond to the variables x_1, x_2, x_3
rows of matrix correspond to the equations

$$A = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 1 & 0 & -3 \\ 2 & 2 & 9 \\ 0 & 1 & 5 \end{bmatrix} \end{array}$$

A is more than a table of numbers. It represents a function

$$f(x_1, x_2, x_3) = f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1x_1 + 0x_2 - 3x_3 \\ 2x_1 + 2x_2 + 9x_3 \\ 0x_1 + 1x_2 + 5x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 2 & 9 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = Ax$$

By definition any function that can be represented in this way, that is by $f(x) = Ax$, is called a linear function.

With this notation

13. $x_1 - 3x_3 = 8$
$2x_1 + 2x_2 + 9x_3 = 7$
$x_2 + 5x_3 = -2$

$\Rightarrow$ becomes $Ax = b$.

Focus: solve the system of equations:

① Does it even have a solution?
② If so how many solutions?
③ How to find the solutions?

$\Bigg\}$ Well posedness of a mathematical problem
exactly one solution

$$\cancel{2x_1} \qquad - 6x_3 = 16$$
$$-\left(\cancel{2x_1} + 2x_2 + 9x_3 = 7 \right)$$

$$\left.\begin{aligned} -2x_2 - 15x_3 &= 9 \\ x_2 + 5x_3 &= -2 \end{aligned}\right\}$$ Two equations in two unknowns... because we eliminated x_1

When eliminating variables we rescale equations, subtract them and choose which ones to work with...

Represent the system $Ax = b$ by the augmented matrix

$$\begin{array}{c} \\ [A|b] = \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & b \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \end{array}$$

$\Big\}$ rows correspond to equations

ELEMENTARY ROW OPERATIONS

1. (Replacement) Replace one row by the sum of itself and a multiple of another row.[1]
   $r_i \leftarrow r_i + \alpha r_j \quad i \neq j$ add α times row j to row i
2. (Interchange) Interchange two rows.
   $r_i \leftrightarrow r_j \quad i \neq j$ switch row i with row j
3. (Scaling) Multiply all entries in a row by a nonzero constant.
   $r_i \leftarrow \alpha r_i \quad \alpha \neq 0$ rescale row i by α.

Interesting fact: Each of the row operations (itself) can be represented by a linear function

Interchange:
$r_1 \leftrightarrow r_2$

$$g\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ x_1 \\ x_3 \end{bmatrix}$$

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1x_1 + 0x_2 - 3x_3 \\ 2x_1 + 2x_2 + 9x_3 \\ 0x_1 + 1x_2 + 5x_3 \end{bmatrix}$$

$$g \circ f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = g\left(\begin{bmatrix} 1x_1 + 0x_2 - 3x_3 \\ 2x_1 + 2x_2 + 9x_3 \\ 0x_1 + 1x_2 + 5x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + 2x_2 + 9x_3 \\ x_1 + 9x_3 \\ x_2 + 5x_3 \end{bmatrix}.$$