

Example 2.1. A manufacturer of color TV sets is planning the introduction of two new products, a 19-inch LCD flat panel set with a manufacturer's suggested retail price (MSRP) of \$339 and a 21-inch LCD flat panel set with an MSRP of \$399. The cost to the company is \$195 per 19-inch set and \$225 per 21-inch set, plus an additional \$400,000 in fixed costs. In the competitive market in which these sets will be sold, the number of sales per year will affect the average selling price. It is estimated that for each type of set, the average selling price drops by one cent for each additional unit sold. Furthermore, sales of the 19-inch set will affect sales of the 21-inch set, and vice-versa. It is estimated that the average selling price for the 19-inch set will be reduced by an additional 0.3 cents for each 21-inch set sold, and the price for the 21-inch set will decrease by 0.4 cents for each 19-inch set sold. How many units of each type of set should be manufactured?

Type of TV	MSRP	Cost	Fixed cost
19"	339	195	400 000
21"	399	225	

x_1 the number of 19" TVs sold

x_2 the number of 21" TVs sold

$$p_1(x_1, x_2) = 339 - 0.01x_1 - 0.003x_2$$

price to sell 19" TVs

$$p_2(x_1, x_2) = 399 - 0.004x_1 - 0.01x_2$$

~~price to sell 21" TVs~~

$$C(x_1, x_2) = 400000 + 195x_1 + 225x_2$$

total cost

$$R(x_1, x_2) = x_1 p_1(x_1, x_2) + x_2 p_2(x_1, x_2)$$

total revenue

$$P(x_1, x_2) = R(x_1, x_2) - C(x_1, x_2)$$

total Profit

```
julia> p1(x1,x2)=339-0.01*x1-0.003*x2
p1 (generic function with 1 method)
```

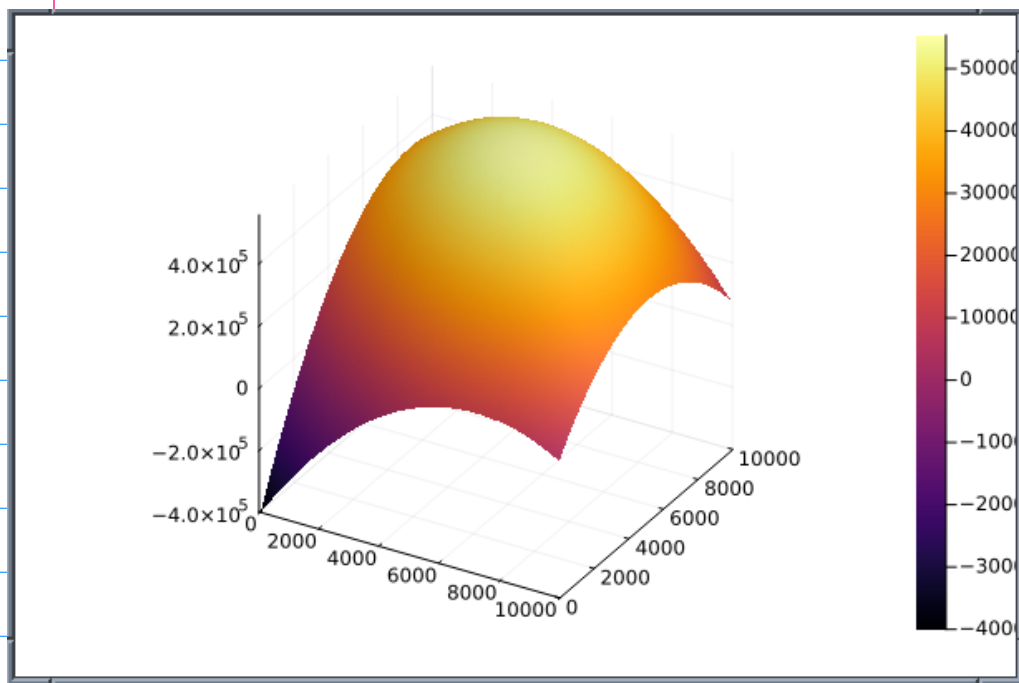
```
julia> p2(x1,x2)=399-0.004*x1-0.01*x2
p2 (generic function with 1 method)
```

```
julia> C(x1,x2)=400_000+195*x1+225*x2
C (generic function with 1 method)
```

```
julia> R(x1,x2)=x1*p1(x1,x2)+x2*p2(x1,x2)
R (generic function with 1 method)
```

```
julia> P(x1,x2)=R(x1,x2)-C(x1,x2)
P (generic function with 1 method)
```

Visualize goes hand in hand with modeling



```
julia> using Plots
```

```
julia> surface(0:100:10000, 0:100:10000, P)
```

Trying to maximize $P(x_1, x_2)$ on the set

$$S = \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0\} \cap \mathbb{Z} \times \mathbb{Z}$$

set of integers

$$\frac{\partial P}{\partial x_1} = \frac{\partial R}{\partial x_1} - \frac{\partial C}{\partial x_1} = \frac{\partial}{\partial x_1} (x_1 p_1 + x_2 p_2) - 195$$

$$= p_1 + x_1 \frac{\partial p_1}{\partial x_1} + x_2 \frac{\partial p_2}{\partial x_1} - 195$$

$$= 339 - 0.01 x_1 - 0.003 x_2 + x_1 (-0.01) + x_2 (-0.004) = 0$$

$$\frac{\partial P}{\partial x_2} = 0$$

$$p_1(x_1, x_2) = 339 - 0.01 x_1 - 0.003 x_2$$

$$p_2(x_1, x_2) = 399 - 0.004 x_1 - 0.01 x_2$$

$$C(x_1, x_2) = 400000 + 195 x_1 + 225 x_2$$

Solve
system

```
julia> using Symbolics
```

```
julia> @variables x1,x2
```

```
2-element Vector{Num}:
```

```
x1
```

```
x2
```

```
julia> P(x1,x2)
```

```
-400000 - 195x1 - 225x2 + x1*(339 - (1//100)*x1 - (3//1000)*x2) + (399 - (1//250)  
) * x1 - (1//100) * x2 * x2
```

```
julia> D(f,x)=expand_derivatives(Differential(x)(f))  
D (generic function with 1 method)
```

```
julia> D(P(x1,x2),x1)
```

```
144 - (1//50)*x1 - (7//1000)*x2
```

```
julia> D(P(x1,x2),x2)
```

```
174 - (7//1000)*x1 - (1//50)*x2
```