Example 2.3. Maximize $x + 2y + 3z$ over the set $x^2 + y^2 + z^2 = 3$. $g(x,y,z) = 3$

$f(x,y,z) = x + 2y + 3z$ and $g(x,y,z) = x^2 + y^2 + z^2$

$\nabla f = \begin{pmatrix} \partial f/\partial x \\ \partial f/\partial y \\ \partial f/\partial z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\quad \nabla g = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$

Condition for a maximum is $\nabla f = \lambda \nabla g$

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$ $\quad$ $2x\lambda = 1$, $2y\lambda = 2$, $2z\lambda = 3$

Solve for x, y, z in terms of λ.

$x = \frac{1}{2\lambda}$, $y = \frac{1}{\lambda}$, $z = \frac{3}{2\lambda}$

Now use the constraint to solve for λ

$g(x,y,z) = 3$ $\quad x^2 + y^2 + z^2 = 3$

$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{\lambda}\right)^2 + \left(\frac{3}{2\lambda}\right)^2 = 3$ $\quad \frac{1}{4} + 1 + \frac{9}{4} = 3\lambda^2$

$\frac{7}{2} = 3\lambda^2$ $\quad \lambda^2 = \frac{7}{6}$ $\quad \lambda = \sqrt{\frac{7}{6}} \approx 1.08$

Now plug λ in to find constrained optimum

```
julia> 1//4+1+9//4
7//2

julia> 7//2/3
7//6
```

$x = \frac{1}{2\lambda}$, $y = \frac{1}{\lambda}$, $z = \frac{3}{2\lambda}$

$x = \frac{1}{2}\sqrt{\frac{6}{7}}$ $\quad y = \sqrt{\frac{6}{7}}$ $\quad z = \frac{3}{2}\sqrt{\frac{6}{7}}$

$\lambda = \sqrt{42}/6$ $\quad \lambda = \sqrt{\frac{7}{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{42}}{6}$

$p_1(x) = 339 - 0.01x_1 - .003x_2$
$p_2(x) = 399 - 0.004x_1 - .01x_2$

$C(x_1,x_2) = 400000 + 195x_1 + 225x_2$
$R(x_1,x_2) = x_1p_1(x_1,x_2) + x_2p_2(x_1,x_2)$
$P(x_1,x_2) = R(x_1,x_2) - C(x_1,x_2)$.

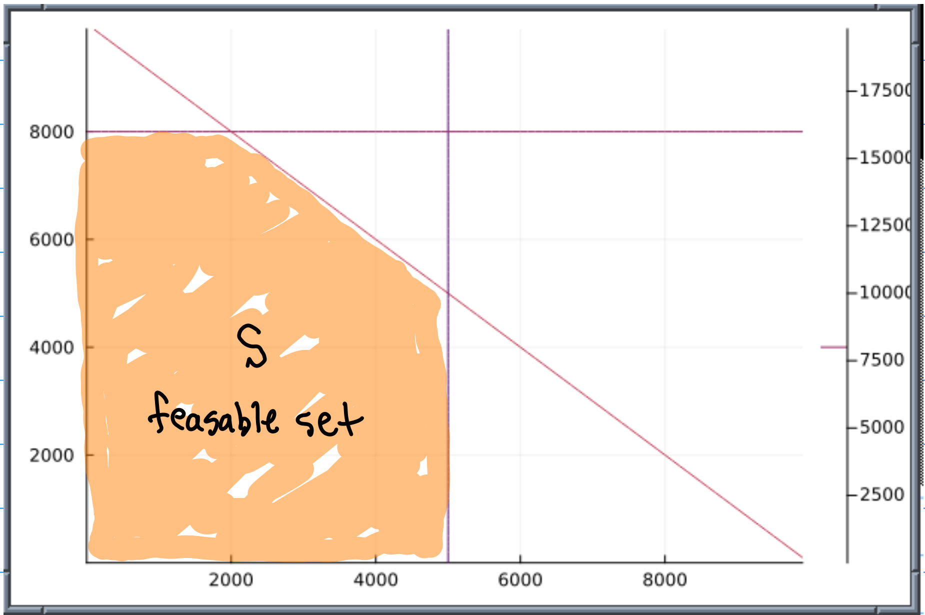
$S = \{(x_1,x_2) : \underbrace{x_1 \geq 0,\ x_2 \geq 0}_{\text{constaints}}\} \cap \mathbb{Z} \times \mathbb{Z}$

more constraints

lines, but the company feels that the new products will be more profitable. It is estimated that the available production capacity will be sufficient to produce 10,000 sets per year (≈ 200 per week). The company has an ample supply of 19–inch and 21–inch LCD panels and other standard components; however, the circuit boards necessary for constructing the sets are currently in short supply. Also, the 19–inch TV requires a different board than the 21–inch model because of the internal configuration, which cannot be changed without a major redesign, which the company is not prepared to undertake at this time. The supplier is able to supply 8,000 boards per year for the 21–inch model and 5,000 boards per year for the 19–inch model. Taking this information into account, how should

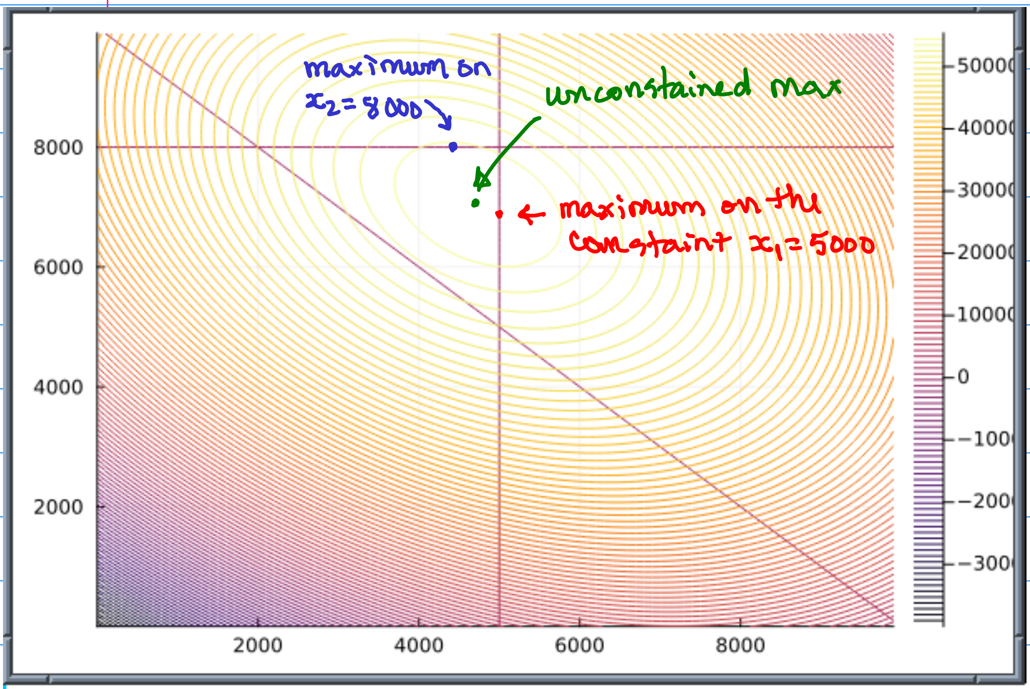
$x_1 + x_2 \leq 10000$ $\quad x_2 \leq 8000$ $\quad x_1 \leq 5000$

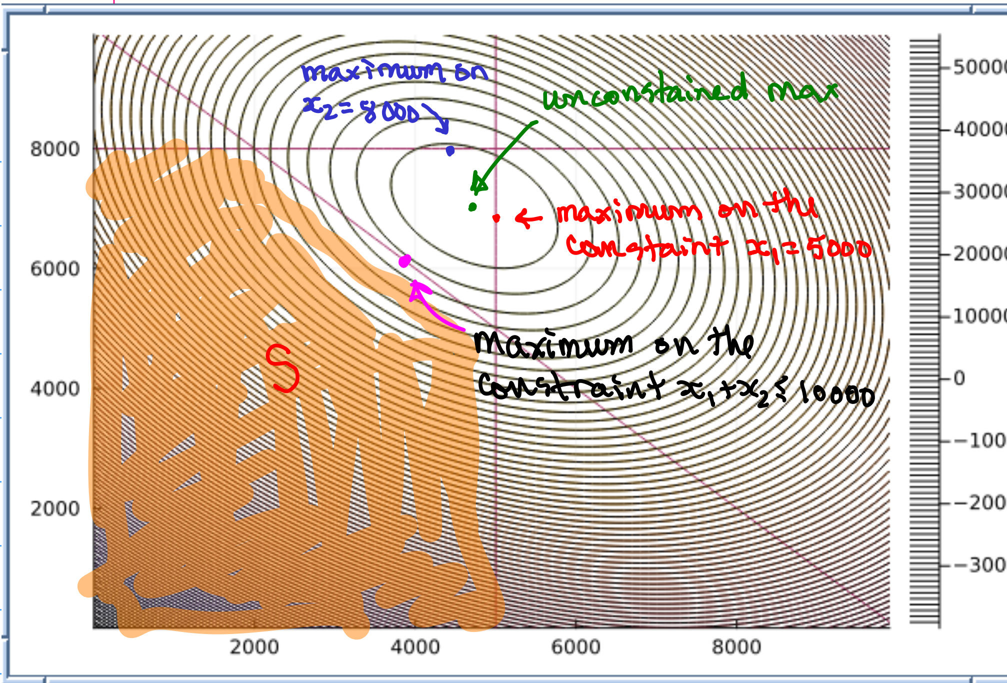
$S = \{(x_1,x_2) : x_1 \geq 0,\ x_2 \geq 0,\ x_1 \leq 5000,\ x_2 \leq 8000,\ x_1+x_2 \leq 10000\} \cap \mathbb{Z} \times \mathbb{Z}$



```
julia> using Plots

julia> g(x1,x2)=x1+x2
g (generic function with 1 method)

julia> xs=1:100:10000
1:100:9901

julia> ys=1:100:10000
1:100:9901

julia> contour(xs,ys,g,levels=[10000])

julia> contour!(xs,ys,(x1,x2)->x1,levels=[5000])
┌ Warning: Multiple series with different levels
│ Colorbar may not reflect all series correctly.
└ @ Plots ~/.julia/packages/Plots/xC48f/src/backe

julia> contour!(xs,ys,(x1,x2)->x2,levels=[8000])
┌ Warning: Multiple series with different levels
│ Colorbar may not reflect all series correctly.
└ @ Plots ~/.julia/packages/Plots/xC48f/src/backe
```

```
 first_axis >= 0 && first_axis + axes
julia> contour!(xs,ys,P,levels=100)
┌ Warning: Multiple series with diffe
│ Colorbar may not reflect all series
└ @ Plots ~/.julia/packages/Plots/xC4
```





$p_1(x) = 339 - 0.01x_1 - .003x_2$

$p_2(x) = 399 - 0.004x_1 - .01x_2$

$C(x_1, x_2) = 400000 + 195x_1 + 225x_2$

$R(x_1, x_2) = x_1 p_1(x_1, x_2) + x_2 p_2(x_1, x_2)$

$P(x_1, x_2) = R(x_1, x_2) - C(x_1, x_2).$

$\nabla C = \begin{bmatrix} 195 \\ 225 \end{bmatrix}$

$\nabla R = \begin{bmatrix} \partial R / \partial x_1 \\ \partial R / \partial x_2 \end{bmatrix}$

$S = \{(x_1, x_2) : x_1 \geq 0, x_2 \geq 0, x_1 \leq 5000, x_2 \leq 8000, x_1 + x_2 \leq 10000\} \cap \mathbb{Z} \times \mathbb{Z}$

$g(x_1, x_2) = x_1 + x_2$

$\nabla g = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\nabla P = \lambda \nabla g$

$\nabla R - \nabla C = \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\frac{\partial R}{\partial x_1} = p_1 + x_1 \frac{\partial p_1}{\partial x_1} + x_2 \frac{\partial p_2}{\partial x_1} = p_1 + x_1(-0.01) + x_2(-.004)$

$\frac{\partial R}{\partial x_2} = x_1 \frac{\partial p_1}{\partial x_2} + p_2 + x_2 \frac{\partial p_2}{\partial x_2} = x_1(-.003) + p_2 + x_2(-.01)$

```
julia> dpdx1=D(P(x1,x2),x1)
144 - (1//50)*x1 - (7//1000)*x2
```
$= \lambda$

```
julia> dpdx2=D(P(x1,x2),x2)
174 - (7//1000)*x1 - (1//50)*x2
```
$= \lambda$

page 38 in the text...

$$144 - 0.02x_1 - 0.007x_2 = \lambda$$

$$174 - 0.007x_1 - 0.02x_2 = \lambda.$$