relative gives non-dimensional ratio.

Sensitivity analysis:

$$S(x_1, a) = \frac{a}{x_1} \cdot \frac{dx_1}{da}$$

$$S(x_2, a) = \frac{a}{x_2} \cdot \frac{dx_2}{da}$$

Model: $a = 0.01$ elasticity of demand for 19" televisions

```
julia> p1(x1,x2)=339-1//100*x1-3//1000*x2
p1 (generic function with 1 method)

julia> p2(x1,x2)=399-4//1000*x1-1//100*x2
p2 (generic function with 1 method)

julia> C(x1,x2)=400_000+195*x1+225*x2
C (generic function with 1 method)

julia> R(x1,x2)=x1*p1(x1,x2)+x2*p2(x1,x2)
R (generic function with 1 method)

julia> P(x1,x2)=R(x1,x2)-C(x1,x2)
P (generic function with 1 method)
```

$$\nabla P = \lambda \nabla g \quad \text{max. condition}$$

$g(x_1, x_2) = x_1 + x_2$

constraint is $g(x_1, x_2) = 10\,000$.

$$\nabla g = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

from text:

$$144 - \underbrace{0.02}_{2a} x_1 - 0.007 x_2 = \lambda$$
$$174 - 0.007 x_1 - 0.02 x_2 = \lambda.$$

$$\nabla P = \begin{bmatrix} \partial P/\partial x_1 \\ \partial P/\partial x_2 \end{bmatrix}$$

$\nabla P = \lambda \nabla g$

becomes

$Ax = b$

```
julia> using Symbolics

julia> @variables a,x1,x2,lambda
4-element Vector{Num}:
      a
     x1
     x2
 lambda

julia> b=[lambda,lambda]
2-element Vector{Num}:
 lambda
 lambda

julia> A=[2a 7//1000; 7//1000 2/100]
2×2 Matrix{Num}:
      2a   7//1000
 7//1000      0.02
```

Handwritten corrections: 144− and 174− before each lambda in b; 2//100 replacing 2/100; 1//50 replacing 0.02.

```
julia> X=A\b
2-element Vector{Num}:
 (174 - lambda + ((-1//50)*(144 - lambda - (2000//7)*a*(174 - lambda))) / ((7//1
000) - (40//7)*a)) / (7//1000)
                                                   (144 - lambda - (2000//7)*a*(174 - lamb
da)) / ((7//1000) - (40//7)*a)

julia> expand(simplify(X[1]))
(-(831//500) + (13//1000)*lambda) / ((49//1000000) - (1//25)*a)
```

$$x_1 = \frac{-\frac{831}{500} + 13/1000\ \lambda}{\frac{49}{1000000} - \frac{1}{25}a}$$

want $\frac{dx_1}{da}$ but note that λ depends on a

```
julia> simplify(expand(simplify(X[2])))
(144 - (348000//7)*a - lambda + (2000//7)*a*lambda) / ((7//1000) - (40//7)*a)
```

$$x_2 = \frac{144 - \frac{348000}{7}a - \lambda\left(1 - \frac{2000}{7}a\right)}{\frac{7}{1000} - \frac{40}{7}a}$$

want $\frac{dx_2}{da}$ but note that λ depends on a

Therefore

$$S(x_1,a) = \frac{a}{x_1}\frac{dx_1}{da} = \frac{a}{x_1}\left(\frac{\partial x_1}{\partial a} + \frac{\partial x_1}{\partial \lambda}\frac{d\lambda}{da}\right)$$

$$= -\frac{a}{x_1}\left[\left(\frac{-\frac{831}{500} + 13/1000\ \lambda}{\left(\frac{49}{1000000} - \frac{1}{25}a\right)^2}\right)\left(\frac{1}{25}\right) + \frac{13}{1000}\,\frac{1}{\frac{49}{1000000} - \frac{1}{25}a}\,\frac{d\lambda}{da}\right]$$

Use the constraint $x_1 + x_2 = 10000$ to solve for λ