

Therefore

$$S(x_1, a) = \frac{a}{x_1} \frac{dx_1}{da} = \frac{a}{x_1} \left( \frac{\partial x_1}{\partial a} + \frac{\partial x_1}{\partial \lambda} \frac{d\lambda}{da} \right)$$

$$= \frac{a}{x_1} \left[ \frac{-\frac{831}{500} + \frac{13}{1000} \lambda}{\left( \frac{49}{1000000} - \frac{1}{25} a \right)^2} \right] \left( \frac{1}{25} \right) + \frac{13}{1000} \frac{1}{49 - \frac{1}{25} a} \frac{d\lambda}{da}$$

evaluate at  $a=0.01$  and the corresponding optimal production level to get the elasticity at that point

Use the constraint  $x_1 + x_2 = 10000$  to solve for  $\lambda$

```
julia> using Symbolics
```

```
julia> @variables a,x1,x2,lambda
```

```
4-element Vector{Num}:  
  a  
  x1  
  x2  
  lambda
```

```
julia> b=[BigInt(144)-lambda, BigInt(174)-lambda]
```

```
2-element Vector{Num}:  
 144 - lambda  
 174 - lambda
```

```
julia> A=[BigInt(2)*a BigInt(7)//1000  
         BigInt(7)//1000 BigInt(2)//100]
```

```
2x2 Matrix{Num}:  
  2a  7//1000  
  7//1000  1//50
```

```
julia> X=A\b
```

```
2-element Vector{Num}:  
 (174 - lambda + ((-1//50)*(144 - lambda - (2000//7)*a*(174 - lambda))) / ((  
 000) - (40//7)*a)) / (7//1000)
```

```
(144 - lambda - (2000//7)*a*(174 -
```

```
da)) / ((7//1000) - (40//7)*a)
```

```
julia> a0=BigInt(1)//100 ←
```

```
1//100
```

```
julia> g(x1,x2)=x1+x2
```

```
g (generic function with 1 method)
```

```

julia> mylambda=Symbolics.solve_for(g(X[1],X[2])-BigInt(10000),lambda)
(-10000 + (174 + ((-1//50)*(144 - (348000//7)*a)) / ((7//1000) - (40//7)*a)) / (7//1000) + (144 - (348000//7)*a) / ((7//1000) - (40//7)*a)) / (-((-1 + (2000//7)*a) / ((7//1000) - (40//7)*a)) - ((-1 + ((-1//50)*(-1 + (2000//7)*a)) / ((7//1000) - (40//7)*a)) / (7//1000))

```

Solve first the production levels when  $a = .01$

$$\lambda = 24$$

optimal production level  $x_1 = 50000/13$

$$x_2 = 80000/13$$

```

julia> lambda0=substitute(mylambda,a=>a0)
24//1

```

```

julia> x10=substitute(X[1],[a=>a0,lambda=>lambda0])
50000//13

```

```

julia> x20=substitute(X[2],[a=>a0,lambda=>lambda0])
80000//13

```

```

julia> p1(x1,x2)=339-a*x1-3//1000*x2
p1 (generic function with 1 method)

```

```

julia> p2(x1,x2)=399-4//1000*x1-1//100*x2
p2 (generic function with 1 method)

```

```

julia> C(x1,x2)=400_000+195*x1+225*x2
C (generic function with 1 method)

```

```

julia> R(x1,x2)=x1*p1(x1,x2)+x2*p2(x1,x2)
R (generic function with 1 method)

```

```

julia> P(x1,x2)=R(x1,x2)-C(x1,x2)
P (generic function with 1 method)

```

```

julia> substitute(P(x10,x20),a=>a0)
6920000//13

```

← maximum profit.

```
julia> pardx1da=expand(simplify(D(X[1],a)))
(- (831//612500) + (13//1225000)*lambda) / ((49//1000000000000) -
(1//12500000)*a + (1//30625)*(a^2))
```

```
julia> pardx1dlambda=expand(simplify(D(X[1],lambda)))
(13//1000) / ((49//1000000) - (1//25)*a)
```

$$S(x_1, a) = \frac{a}{x_1} \frac{dx_1}{da} = \frac{a}{x_1} \left( \frac{\partial x_1}{\partial a} + \frac{\partial x_1}{\partial \lambda} \frac{d\lambda}{da} \right)$$

```
julia> dlambdada=D(simplify(expand(mylambda)),a)
((-2//49)*((143//6125) - (52//49)*a)) / (((3//24500) + (2//49)*a
)^2) + (-52//49) / ((3//24500) + (2//49)*a)
```

```
julia> tmp1=pardx1da+pardx1dlambda*dlambdada
((13//1000)*((-2//49)*((143//6125) - (52//49)*a)) / (((3//24500
) + (2//49)*a)^2) + (-52//49) / ((3//24500) + (2//49)*a)) / ((4
9//1000000) - (1//25)*a) + (- (831//612500) + (13//1225000)*lambda
a) / ((49//1000000000000) - (1//12500000)*a + (1//30625)*(a^2))
```

```
julia> Sx1a=substitute(a/x1*tmp1,
[a=>a0, lambda=>lambda0, x1=>x10])
-10//13
```

```
julia> -10/13
-0.7692307692307693
```

$$S(x_1, a) = -\frac{10}{13} \approx -0.77$$

```

julia> pardx2da=expand(simplify(D(X[2],a)))
(- (831//12250) + (13//24500)*lambda) / (- (7//1000000000) + (1//87500)*a - (8//1715)*(a^2))

julia> pardx2dlambda=expand(simplify(D(X[2],lambda)))
(-1 + (2000//7)*a) / ((7//1000) - (40//7)*a)

julia> tmp2=pardx2da+pardx2dlambda*dlambda
((-1 + (2000//7)*a)*((( -2//49)*((143//6125) - (52//49)*a)) / (((3//24500) + (2//49)*a)^2) + (-52//49) / ((3//24500) + (2//49)*a))) / ((7//1000) - (40//7)*a) + (- (831//12250) + (13//24500)*lambda) / (- (7//1000000000) + (1//87500)*a - (8//1715)*(a^2))

julia> Sx2a=substitute(a/x2*tmp2,
[a=>a0, lambda=>lambda0, x2=>x20])
25//52

```

```

julia> 25/52
0.4807692307692308

```

$$S(x_1, a) = \frac{dx_1}{da} \cdot \frac{a}{x_1} = -0.77$$

$$S(x_2, a) = \frac{dx_2}{da} \cdot \frac{a}{x_2} = 0.48.$$

Text

$$\nabla P = \lambda \nabla g = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix}$$

What about profit..

$$S(P, a) = \frac{a}{P} \frac{dP}{da} = \frac{a}{P} \left( \frac{\partial P}{\partial x_1} \frac{dx_1}{da} + \frac{\partial P}{\partial x_2} \frac{dx_2}{da} + \frac{\partial P}{\partial a} \right)$$

$$= \frac{a}{P} \left( \lambda \frac{dx_1}{da} + \lambda \frac{dx_2}{da} + \frac{\partial P}{\partial a} \right)$$

```

julia> P0=substitute(P(x10,x20),a=>a0)
6920000//13

```

```

julia> tmpSPa=a0/P0*(lambda0*tmp1+lambda0*tmp2+pardpda)
(13//692000000)*((24//1)*(((13//1000)*((( -2//49)*((143//6125) - (52//49)*a)) / (((3//24500) + (2//49)*a)^2) + (-52//49) / ((3//24500) + (2//49)*a)))) / ((49//1000000) - (1//25)*a) + (- (831//612500) + (13//1225000)*lambda) / ((49//1000000000000) - (1//12500000)*a + (1//30625)*(a^2))) + (24//1)*((-1 + (2000//7)*a)*((( -2//49)*((143//6125) - (52//49)*a)) / (((3//24500) + (2//49)*a)^2) + (-52//49) / ((3//24500) + (2//49)*a)))) / ((7//1000) - (40//7)*a) + (- (831//12250) + (13//24500)*lambda) / (- (7//1000000000) + (1//87500)*a - (8//1715)*(a^2))) - (x1^2)

```

```

julia> SPa=substitute(tmpSPa,[a=>a0, lambda=>lambda0, x1=>x10])
-625//2249

```