

$$\begin{bmatrix} Y_{n+1,1} \\ Y_{n+1,2} \end{bmatrix} = Y_{n+1} = D Y_n = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} Y_{n,1} \\ Y_{n,2} \end{bmatrix} = \begin{bmatrix} \lambda_1 Y_{n,1} \\ \lambda_2 Y_{n,2} \end{bmatrix}$$

$$Y_{n+1,1} = \lambda_1 Y_{n,1}$$

$$Y_{n+1,2} = \lambda_2 Y_{n,2}$$

$$Y_{n,1} = \lambda_1^{n-1} Y_{1,1}$$

$$Y_{n,2} = \lambda_2^{n-1} Y_{1,2}$$

solution...

recall, ...

$$Y_n = U^{-1} X_n$$

$$X_n = \begin{bmatrix} V_n \\ V_{n-1} \end{bmatrix}$$

$$X_1 = \begin{bmatrix} -V_0 + c a_0 - w k_b V_0 \\ V_0 \end{bmatrix}$$

$$Y_n = \begin{bmatrix} \lambda_1^{n-1} Y_{1,1} \\ \lambda_2^{n-1} Y_{1,2} \end{bmatrix} = \begin{bmatrix} \lambda_1^{n-1} & 0 \\ 0 & \lambda_2^{n-1} \end{bmatrix} Y_1 = \begin{bmatrix} \lambda_1^{n-1} & 0 \\ 0 & \lambda_2^{n-1} \end{bmatrix} U^{-1} \begin{bmatrix} -V_0 + c a_0 - w k_b V_0 \\ V_0 \end{bmatrix}$$

$$X_n = U Y_n = U \begin{bmatrix} \lambda_1^{n-1} & 0 \\ 0 & \lambda_2^{n-1} \end{bmatrix} U^{-1} \begin{bmatrix} -V_0 + c a_0 - w k_b V_0 \\ V_0 \end{bmatrix}$$

only place where n appears...

In order for the spaceship to stabilize and stop moving around we need $|\lambda_1| < 1$ and $|\lambda_2| < 1$.

If $|\lambda_1| < 1$ but $|\lambda_2| \geq 1$ then it could happen that the initial condition is aligned along the eigenvector for λ_1 and the corresponding part for λ_2 is zero. In that case the fact that $|\lambda_2|^n \rightarrow 0$ as $n \rightarrow \infty$ doesn't matter since that part of the solution started out as zero... However any errors in the model would make this part of the solution not quite zero in which case it could grow...

let's set $c = 5$ and $w = 10$ and vary k_0 to see when the eigenvalues have magnitudes larger or smaller than 1.