Two types of dynamical systems

① Continuous

nonlinear

$$\frac{dX}{dt} = G(X)$$

linear

$$\boxed{\frac{dX}{dt} = AX}$$

If eigenvalues of A are negative then $\|X(t)\| \to 0$ as $t \to \infty$

② discrete

non linear

$$X_{n+1} = G(X_n)$$

linear

$$X_{n+1} = AX_n$$

If the magnitude of the eigenvalue's of A are less than 1 then $\|X_n\| \to 0$ as $n \to \infty$

Consider the linear continuous case more carefully...

$$G_L(X) = G(X_0) + DG(X_0)(X - X_0)$$

$$\frac{dX}{dt} = G_L(X) = b + A(X - X_0)$$

$$= \underline{b + AX - AX_0}$$

$$= \underbrace{(b - AX_0)} + AX$$

$$\frac{dX}{dt} = c + AX = AA^{-1}c + AX = A(A^{-1}c + X)$$

$$\frac{d}{dt} A^{-1}c = 0 \qquad \text{since } A^{-1}c \text{ is constant}$$

$$\frac{d(A^{-1}c + X)}{dt} = A(\underbrace{A^{-1}c + X}_{y})$$

$$\boxed{\frac{dY}{dt} = AY.}$$ ← consider this...

Suppose A has eigenvalues λ_i and eigenvectors u_i

$U = [u_1 \vdots u_2]$ (also works with higher dimensions)

$$AU = [\lambda_1 u_1 \vdots \lambda_2 u_2] = [u_1 \vdots u_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = UD$$

$A = UDU^{-1}$

use this to choose my eigenvalues and eigenvectors for A.

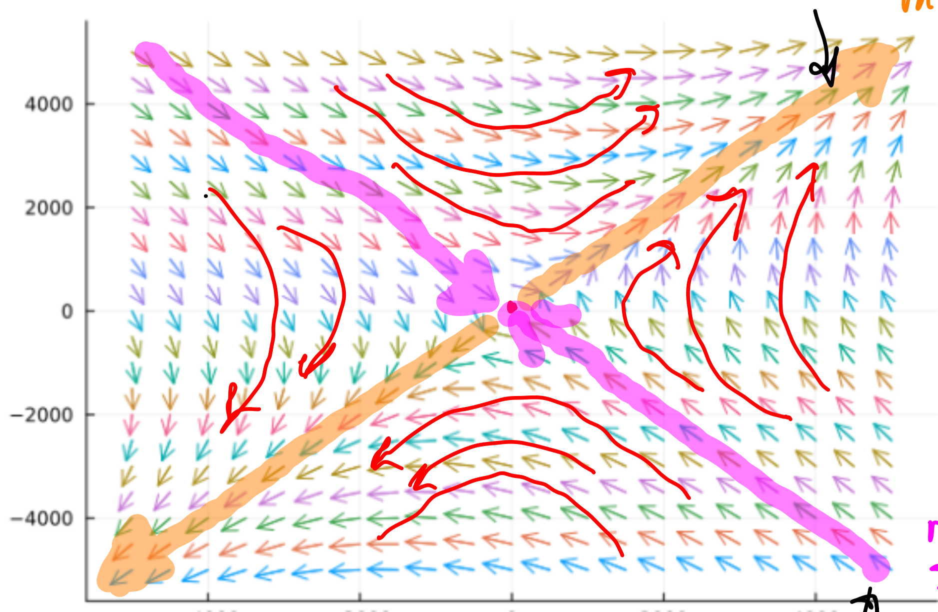
$U = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$ $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -2 \boxed{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

unstable manifold...

move away from fixed point (0,0) along (1,1) since the eigenvalue was positive.

Saddle



move toward (0,1) since eigenvalue −2 is negative...

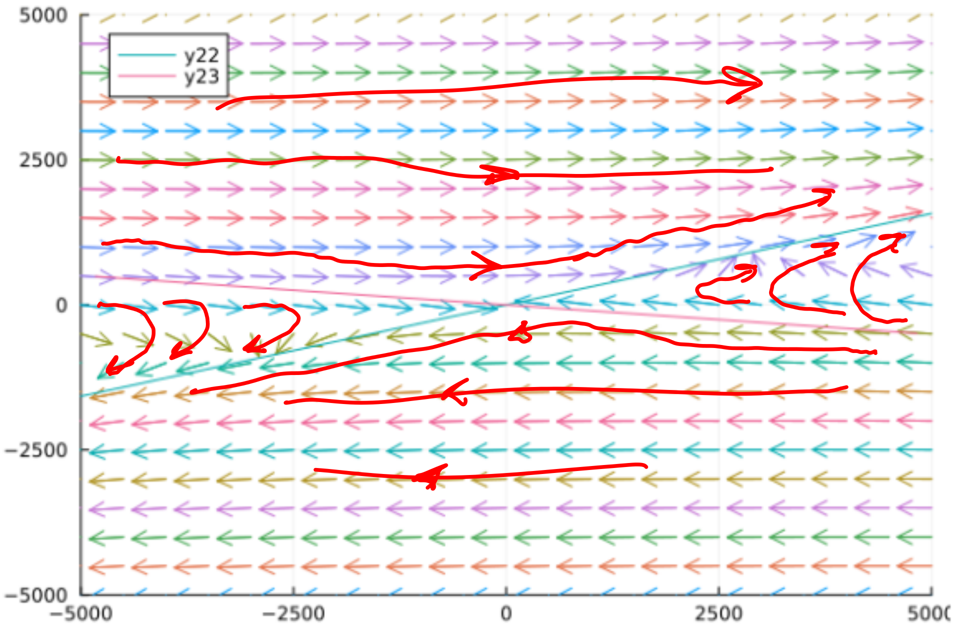
stable manifold

```
U=randn(2,2)
```

```
2×2 Matrix{Float64}:
 -1.3522     -0.965533
 -0.425913    0.0980373
```

```
D=[1 0; 0 -2]
```

```
2×2 Matrix{Int64}:
 1   0
 0  -2
```



If the eigenvalues of $A \in \mathbb{R}^2$ are complex then $\lambda_1 = \overline{\lambda_2}$.

Also we can take the eigenvectors to be complex conjugates as well...

$$D = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \qquad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$