$$X_{n+1} = \begin{bmatrix} v_{n+1} \\ v_n \end{bmatrix} = \begin{bmatrix} v_n(1-wk) - ck\,v_{n-1} \\ v_n \end{bmatrix} = \begin{bmatrix} 1-wk & -ck \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_n \\ v_{n-1} \end{bmatrix}$$

Therefore $X_{n+1} = A X_n$

Find eigenvalues of A. $\det(A - \lambda I) = 0$

$$\det \begin{bmatrix} 1-wk-\lambda & -ck \\ 1 & -\lambda \end{bmatrix} = (1-wk-\lambda)(-\lambda) + ck = 0$$

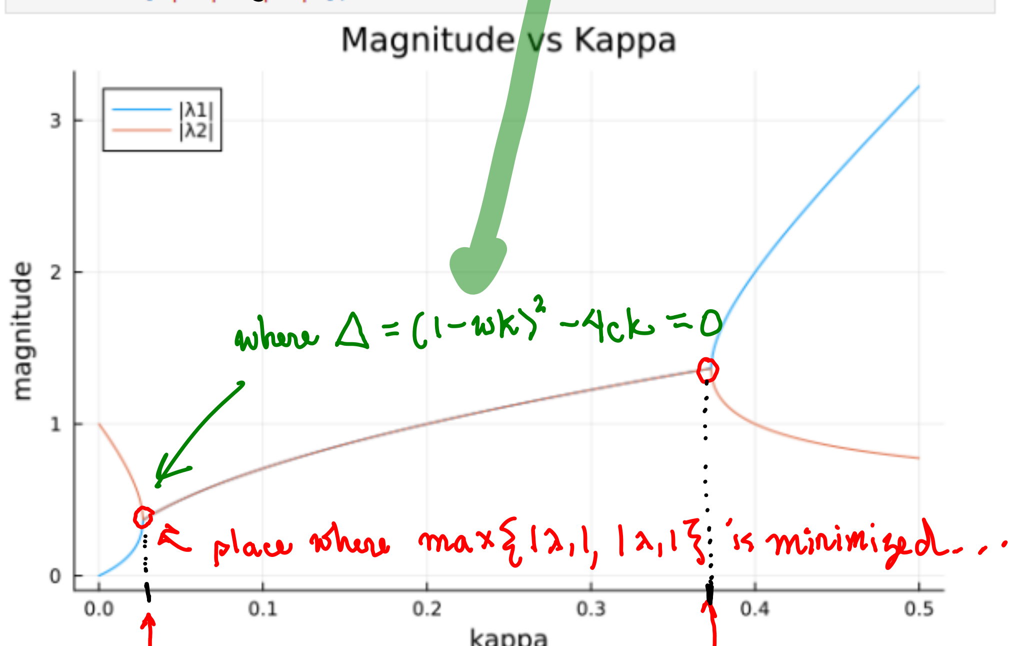
$$\lambda^2 - \lambda(1-wk) + ck = 0$$

$\alpha = 1$ $\beta = -(1-wk)$ $\gamma = ck$

$$\lambda = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} = \frac{1-wk}{2} \pm \frac{\boxed{\sqrt{(1-wk)^2 - 4ck}}}{2} = \lambda_1, \lambda_2$$

$c = 5$ $w = 10$

Out[17]:



where $\Delta = (1-wk)^2 - 4ck = 0$

place where $\max\{|\lambda_1|, |\lambda_1|\}$ is minimized...

0.026794919243112253

0.3732050807568878

$$(1-wk)^2 - 4ck = 0$$

$$1 - 2wk + w^2k^2 - 4ck = 0$$

$$w^2k^2 - (2w + 4c)k + 1 = 0$$

$\alpha = w^2$ $\beta = -(2w + 4c)$ $\gamma = 1$

$$k = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} = \frac{2w + 4c \pm \sqrt{(2w+4c)^2 - 4w^2}}{2w^2}$$

$$(2w+4c)^2 - 4w^2 = 4w^2 + 16wc + 16c^2 - 4w^2$$

$$K = \frac{2w + 4c \pm \sqrt{16wc + 16c^2}}{2w^2} = \frac{w + 2c \pm 2\sqrt{wc + c^2}}{w^2}$$