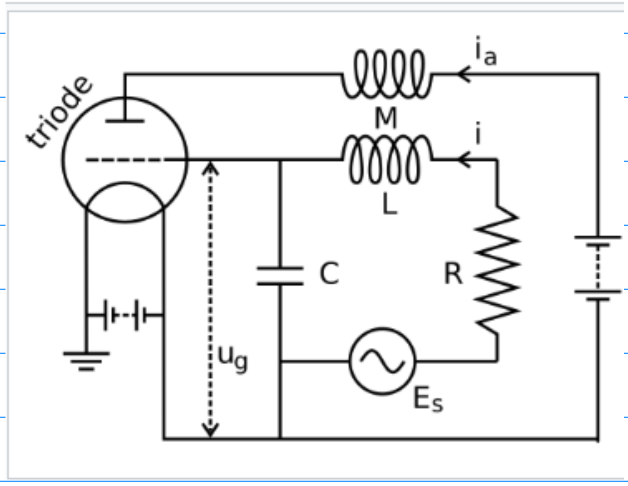


RLC circuit where $v_R = i_R^3 - i_R = f(i_R)$

injects energy linear way

removes energy nonlinearly.

$$\begin{cases} C\dfrac{dv_c}{dt} = i \\ L\dfrac{di}{dt} = -v_c - f(i) = -v_c - i^3 + i \end{cases}$$

$$X = \begin{bmatrix} i \\ v_c \end{bmatrix} \qquad F\left(\begin{bmatrix} i \\ v_c \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{L}(-v_c - i^3 + i) \\ i/C \end{bmatrix}$$

$$\frac{dX}{dt} = F(X)$$

what are the fixed points

$$\frac{1}{L}(-v_c - i^3 + i) = 0 \qquad \text{so} \quad v_c = 0$$

$$i/C = 0 \qquad i = 0$$

one fixed point at the origin.

Linearize about fixed point

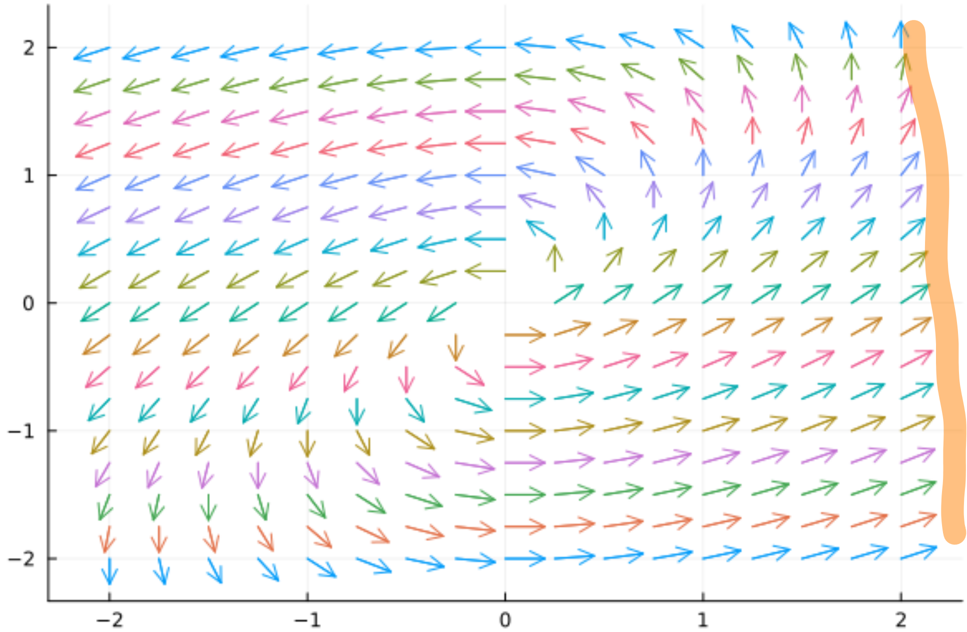
$$DF = \begin{bmatrix} \partial F_1/\partial i & \partial F_1/\partial v_c \\ \partial F_2/\partial i & \partial F_2/\partial v_c \end{bmatrix} = \begin{bmatrix} (1-3i^2)/L & -1/L \\ 1/C & 0 \end{bmatrix}$$

$L=1$

$C=1$

$$DF(0,0) = \begin{bmatrix} 1/L & -1/L \\ 1/C & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

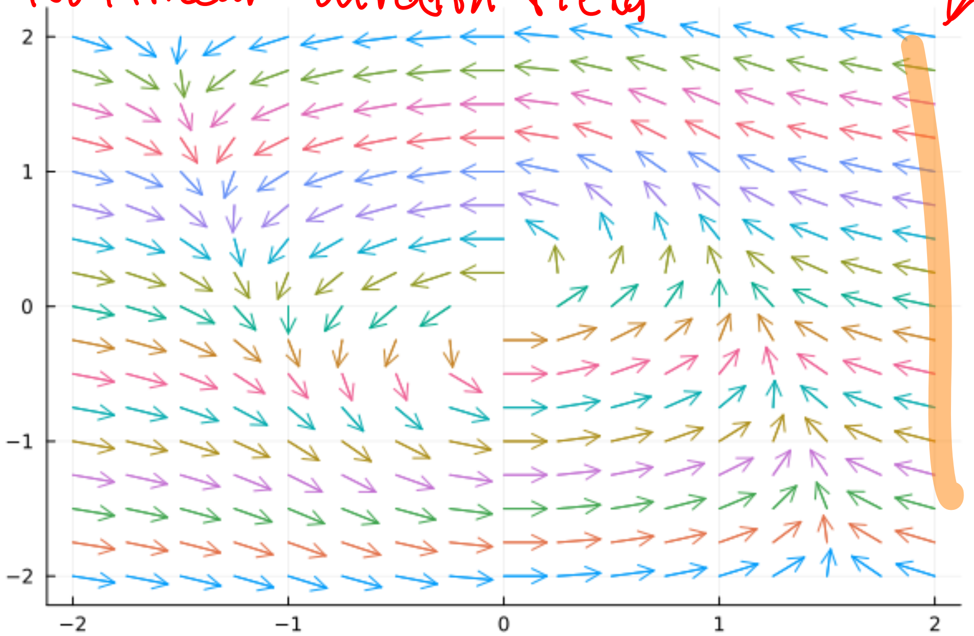
Linear direction field



here solutions grow exp. in time as $t \to \infty$

arrows point in different directions

Non linear direction field

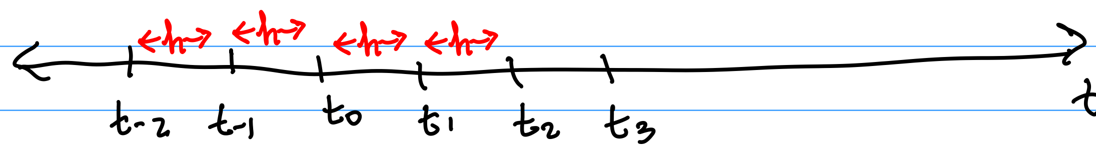


non-linear case solutions do not grow exp. as $t \to \infty$.

How to compute solutions of a nonlinear PDE.

$$\frac{dX}{dt} = F(x)$$

Idea discretize time

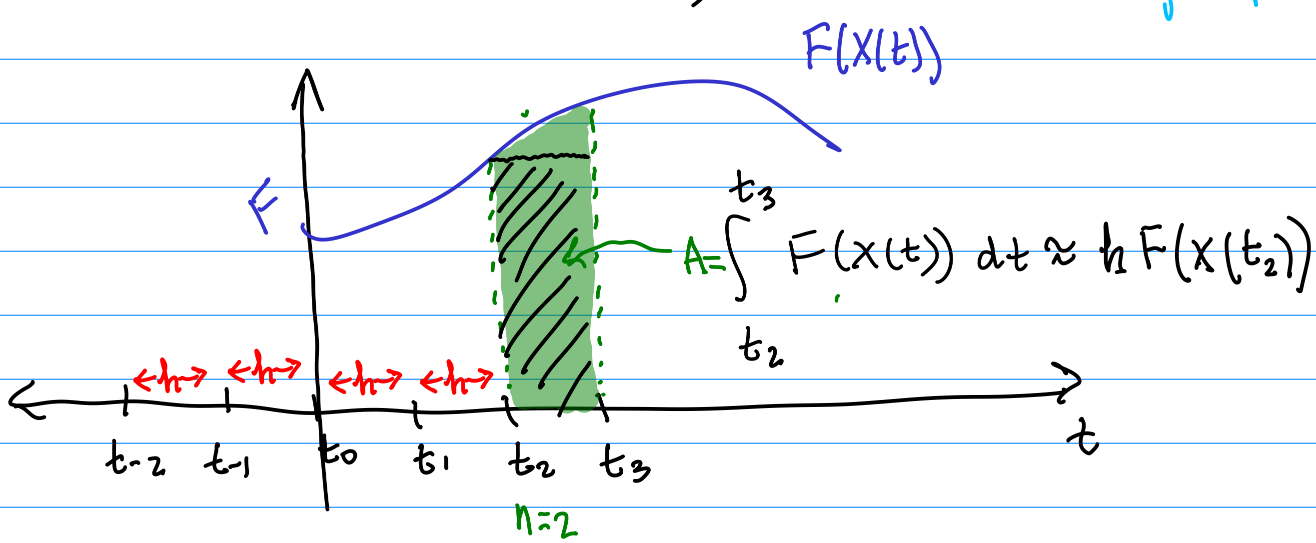


Let $t_n = t_0 + nh$ where h is the gridsize

$$\int_{t_n}^{t_{n+1}} \frac{dX}{dt}\, dt = \int_{t_n}^{t_{n+1}} F(x(t))\, dt$$

$$X(t_{n+1}) - X(t_n) \approx hF(X(t_n))$$

approximation for X evaluated at the grid points in terms of X evaluated at the grid points



$$X(t_{n+1}) - X(t_n) \approx hF(X(t_n))$$

Let $X_n \approx X(t_n)$ let X_n be an approx for the true solution at the grid point

$$X_{n+1} - X_n \approx hF(X_n)$$

Exact relation between approximations

Euler's Explicit Method

$$\boxed{X_{n+1} = X_n + hF(X_n)}$$