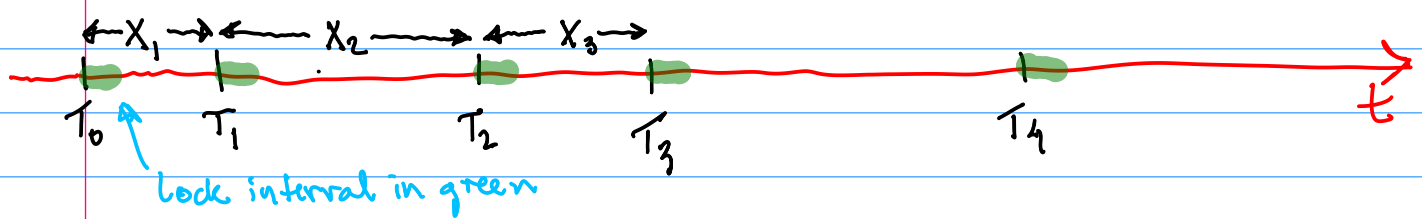
Example 7.3. A "type I counter" is used to measure the radioactive decay in a sample of fissionable material. Decays occur at random, at an unknown rate, and the purpose of the counter is to measure the decay rate. Each radioactive decay locks the counter for a period of 3×10^{-9} seconds, during which time any decays that occur are not counted. How should the data received from the counter be adjusted to account for the lost information?

Model of radio active decay...

What is the data



Let T_n be the times at which the "type I counter" detects a radiating particle.

Let $X_n = T_n - T_{n-1}$ the time between detected particles.

Average time between detected particles.

$$\frac{X_1 + X_2 + \cdots + X_n}{n}$$

Idea is that radiation occurs at a certain rate λ and each radiocative emission is an independent event,

Times between particle emmision theoretically follows a exponential distribution...

Continuous probability distributions

Given a random variable X consider $P(X \leq t) = F(t)$

cumulative distribution function of X.

$$P(a < X \leq b) = P(X \leq b) - P(X \leq a) = F(b) - F(a)$$

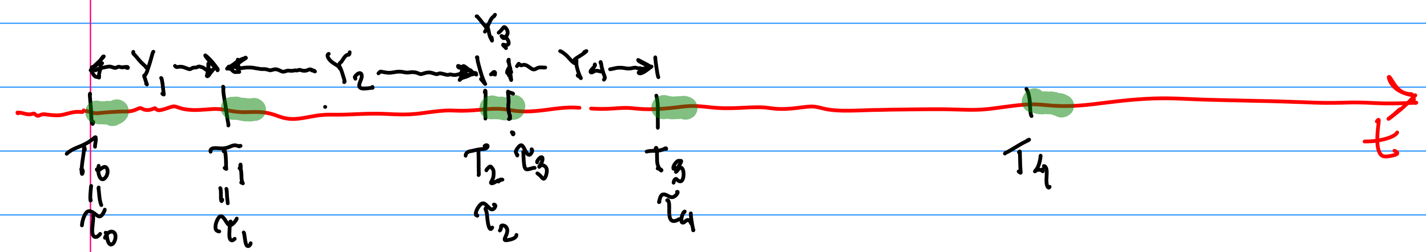
assume → Suppose the function $F(t)$ is differentiable.

Define the density $f(t) = F'(t)$.

$$F(b) - F(a) = \int_a^b F'(t)\,dt = \int_a^b f(t)\,dt$$

Thus

$$P(a < X \leq b) = \int_a^b f(t)\,dt$$



If there wasn't a lock time then maybe more particles were emitted and Y_n is the time between the actual particles emitted.

If particles are emitted with rate λ then

$$G(t) = P(Y_n \leq t) = \begin{cases} 1 - e^{-\lambda t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

assume $t \geq 0$

cumulative distribution function for the exponential distribution.

$$g(t) = G'(t) = \frac{d}{dt}\left(1 - e^{-\lambda t}\right) = \lambda e^{-\lambda t}$$

Let Y be distributed by exp. dist. with rate λ

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \qquad A = \{Y > s+t\} \qquad B = \{Y > s\}$$

$$P(Y > s+t \mid Y > s) = \frac{P(Y > s+t \text{ and } Y > s)}{P(Y > s)}$$

$$= \frac{P(Y > s+t)}{P(Y > s)} = \frac{1 - P(Y \le s+t)}{1 - P(Y \le s)}$$

$$= \frac{\cancel{1} - (\cancel{1} - e^{-\lambda(s+t)})}{\cancel{1} - (\cancel{1} - e^{-\lambda s})} = \frac{\cancel{e^{-\lambda s}}\, e^{-\lambda t}}{\cancel{e^{-\lambda s}}}$$

$$= e^{-\lambda t} = P(Y > t)$$