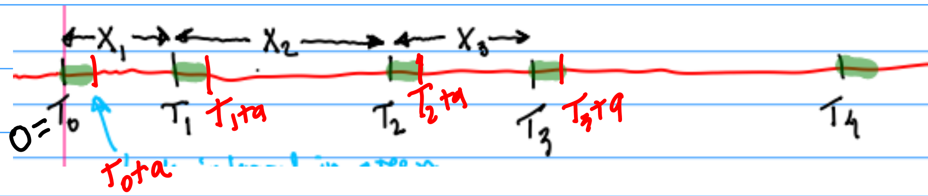
$$P(Y_n \leq t) = \begin{cases} 1 - e^{-\lambda t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



$$P(X_{r_i} \leq t) = \begin{cases} 0 & \text{for } t \leq a \\ P(Y \leq t \mid Y > a) & \text{for } t > a \end{cases}$$

exponentially distributed

$$P(Y \leq t \mid Y > a) = \frac{P(Y \leq t \text{ and } Y > a)}{P(Y > a)}$$

$$P(Y > a) = 1 - P(Y \leq a) = 1 - (1 - e^{-\lambda a}) = e^{-\lambda a}$$

$$P(Y \leq t \text{ and } Y > a) = P(a < Y \leq t) = \int_a^t \lambda e^{-\lambda s}\, ds$$

$$= G(t) - G(a) = (\cancel{1} - e^{-\lambda t}) - (\cancel{1} - e^{-\lambda a})$$

$$= e^{-\lambda a} - e^{-\lambda t}$$

$$P(Y \leq t \mid Y > a) = \frac{e^{-\lambda a} - e^{-\lambda t}}{e^{-\lambda a}} = 1 - e^{-\lambda (t-a)}$$

$$P(X_{r_i} \leq t) = \begin{cases} 0 & \text{for } t \leq a \\ 1 - e^{-\lambda (t-a)} & \text{for } t > a \end{cases}$$

$$P(Y_n \le t) = \begin{cases} 1 - e^{-\lambda t} & \text{for } t \ge 0 \\ 0 & \text{for } t < 0 \end{cases}$$

$$F(t) = P(X_n \le t) = \left\{ \begin{array}{ll} 0 & \text{for } t \le a \\ 1 - e^{-\lambda(t-a)} & \text{for } t > a \end{array} \right\} = P(Z_n \le t - a)$$

where Z_n is an exponentially distributed random variable.

$P(\underbrace{X_n - a}_{Z_n} \le t - a)$

so $Z_n = X_n - a$

or $X_n = Z_n + a$ where Z_n is

Given some data: T_i's then compute $X_i = T_i - T_{i-1}$ and find an average detection interval

$$\text{Avg} = \frac{X_1 + X_2 + \cdots + X_n}{n} \longrightarrow E[X] \quad \text{as } n \to \infty$$

law of large numbers: If you same a random process and take the average of the samples then that averge converges to the expected value of the distribution.

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$F(t) = P(X_n \le t) = \begin{cases} 0 & \text{for } t \le a \\ 1 - e^{-\lambda(t-a)} & \text{for } t > a \end{cases}$$

$$f(t) = F'(t) = \begin{cases} 0 & \text{for } t \le a \\ \lambda e^{-\lambda(t-a)} & \text{for } t > a \end{cases}$$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx = \int_a^{\infty} x \lambda e^{-\lambda(x-a)}\,dx$$

$t = x - a$ so $dt = dx$

$x = t + a$

$$= \int_0^{\infty} (t+a)\lambda e^{-\lambda t}\,dt = \int_0^{\infty} t\lambda e^{-\lambda t}\,dt + a\int_0^{\infty} \lambda e^{-\lambda t}\,dt$$

$$\int_0^{\infty} \lambda e^{-\lambda t}\,dt = -e^{-\lambda t}\Big|_0^{\infty} = \lim_{t\to\infty} -e^{-\lambda t} + 1 = 1$$

use integration by parts

$$\int_0^{\infty} t\lambda e^{-\lambda t}\,dt = t(-e^{-\lambda t})\Big|_0^{\infty} + \int_0^{\infty} e^{-\lambda t}\,dt$$

$u = t \qquad dv = \lambda e^{-\lambda t}dt$

$du = dt \qquad v = -e^{-\lambda t}$

$$= \lim_{t\to\infty} t(-e^{-\lambda t}) - 0(-e^{-\lambda 0}) + -\frac{1}{\lambda}e^{-\lambda t}\Big|_0^{\infty} = \frac{1}{\lambda}$$

Therefore

$$E[X] = \frac{1}{\lambda} + a \quad \text{and} \quad \underbrace{\frac{X_1 + X_2 + \cdots + X_n}{n}}^{T_n \text{ by definition of what the } X_i\text{'s were}} \longrightarrow E[X]$$

If n is large enough, then

$$\frac{1}{\lambda} + a \approx \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{T_n}{n}$$

From this approximation we can estimate λ.

$$\frac{1}{\lambda} = \frac{T_n}{n} - a = \frac{T_n - na}{n} \qquad \lambda = \frac{n}{T_n - na}$$

estimator of the rate of radioactive decay.

Relative sensitivity $S(\lambda, a) = \frac{a}{\lambda}\frac{d\lambda}{da}\Big|_{\lambda = \frac{n}{T_n - na}}$

$$\frac{d\lambda}{da} = \frac{d}{da}\frac{n}{T_n - na} = \frac{-n}{(T_n - na)^2}\frac{d}{da}(T_n - na) = \frac{-n}{(T_n - na)^2}(-n)$$

$$= \frac{n^2}{(T_n - na)^2} = \left(\frac{n}{T_n - na}\right)^2 = \lambda^2$$

by luck to simplify writing.

also depends on rate of decay.

$$S(\lambda, a) = \frac{a}{\lambda}\frac{d\lambda}{da} = \frac{a}{\lambda}\lambda^2 = a\lambda\Big|_{\lambda = \frac{n}{T_n - na}} = a\left(\frac{n}{T_n - na}\right)$$

sensitivity depends on a. Can't do anything about that.

This is also the expected number of decays during the lock time. We can therefore get a better estimate of λ (in relative terms) for a less intensely radioactive source. One simple way to achieve this is to use fewer grams of radioactive material in our sample. Another important source of potential error comes from