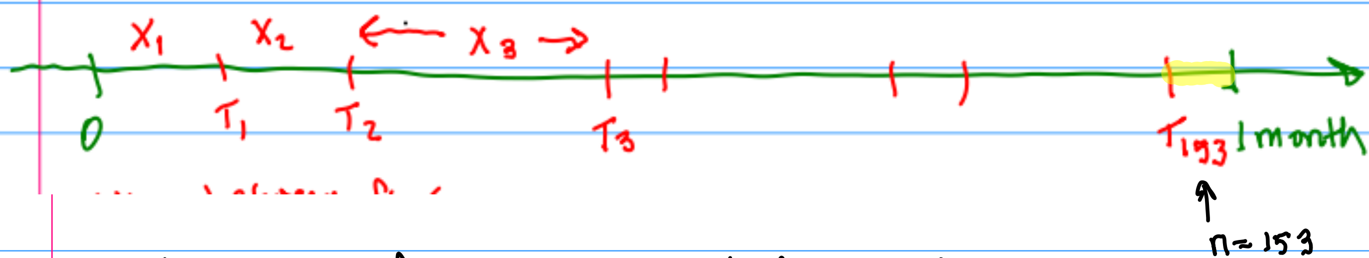
Example 7.4. An emergency 911 service in a local community received an average of 171 calls per month for house fires over the past year. On the basis of this data, the rate of house fire emergencies was estimated at 171 per month. The next month only 153 calls were received. Does this indicate an actual reduction in the rate of house fires, or is it simply a random fluctuation?

rate of fires $\lambda = 171$ per month.



Assumption the X_i's follow an exponential dist with $\lambda = 171$.

H_0: The observed data does not indicate a significant reduction in the rate of house fires.

H_1: The observed data indicates a significant reduction in the rate of house fires.

So there is a random process generating the fires and this leads to another random variable N the counts the fires in a month.

$P(N < 153)$

The probability that (if the assumption that $\lambda = 171$ is true) that there might have been even fewer fires.

Intuitely if the probability of obseving an even more extreme reduction in fires is a very small probability then the observed number of fires 153 is not very consistent with the assumption $\lambda = 171$.

Rule of thumb: If $P(N < 153) \geq 0.05 = 5\%$ then it's not very small.

Goal to calculate this.

complicated random variable

Note that whatever N is that T_N is time of the last fire in the month and it's reasonable to assume $T_N \approx 1$

Eliminate N using this:

Note that $N<153$ means that $T_{153}>1$, that is that the 153 fire occurred after the month was over.

Thus

$$P(N<153) = P(T_{153}>1) = P\left(\frac{T_{153}}{153}-\mu > \frac{1}{153}-\mu\right)$$

$$= P\left(\frac{T_{153}-153\mu}{\sigma\sqrt{153}} > \underbrace{\frac{\sqrt{153}}{\sigma}\left(\frac{1}{153}-\mu\right)}_{\varepsilon}\right)$$

$$\varepsilon = \frac{1-153\mu}{\sigma\sqrt{153}}$$

$$= 1-P\left(\frac{T_{153}-153\mu}{\sigma\sqrt{153}} \le \varepsilon\right) \approx 1-\Phi(\varepsilon)$$

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julia> using SpecialFunctions

julia> lambda=171
171

julia> mu=1/lambda
0.005847953216374269

julia> sigma=1/lambda
0.005847953216374269

julia> epsilon=(1-153*mu)/(sigma*sqrt(153))
1.4552137502179994

julia> Phi(epsilon)=1/2*erfc(-epsilon/sqrt(2))
Phi (generic function with 1 method)

julia> 1-Phi(epsilon)
0.07280504769843332
```

greater than 5% so the observation does not provide evidence at the 95% significance level that the rate of fires has decreased.

H_0: The observed data does not indicate a significant change in the rate of house fires.

H_1: The observed data indicates a significant change in the rate of house fires.

put absolute values in what we had before

$$P\left(\frac{|T_{153}-153\mu|}{\sigma\sqrt{153}} > \underbrace{\frac{\sqrt{153}}{\sigma}\left|\frac{t}{153}-\mu\right|}_{\varepsilon}\right)$$

$$\varepsilon = \frac{|t-153\mu|}{\sigma\sqrt{153}}$$

$$= 1 - P\left(\frac{|T_{153}-153\mu|}{\sigma\sqrt{153}} \leq \varepsilon\right) \approx 1 - \Phi(\varepsilon) + \Phi(-\varepsilon)$$

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julia> 1-Phi(epsilon)+Phi(-epsilon)
0.14561009539686665
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again the probability is big that a more extreme observation could be made under the assumption $\lambda = 171$. So there is no reason to believe any significant change in the rate of fires...