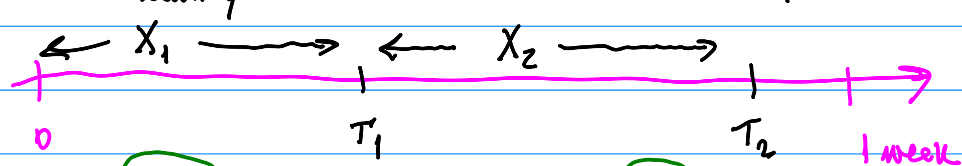
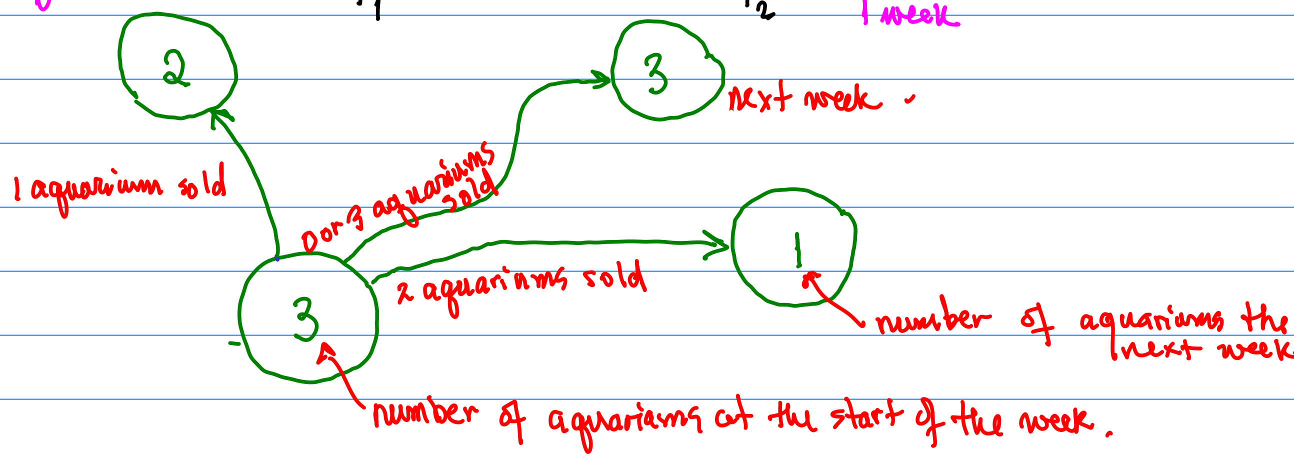
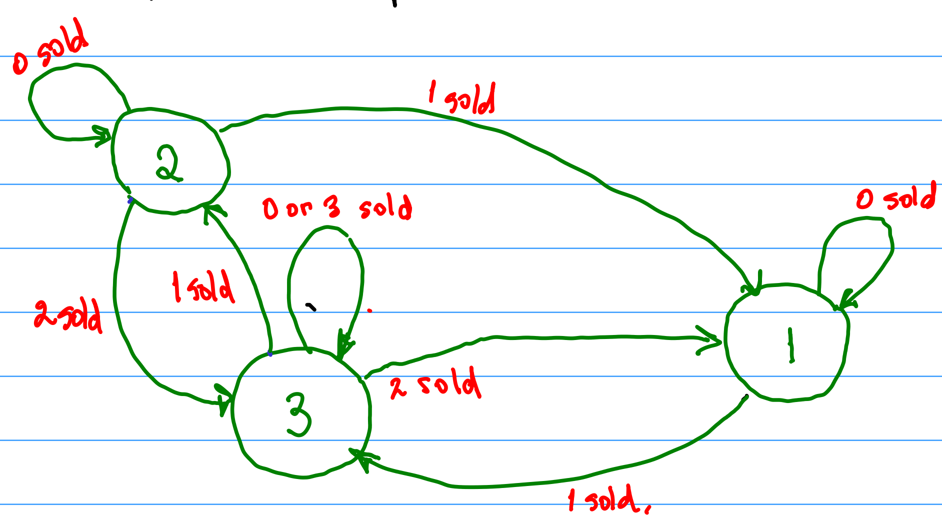
Example 8.1. A pet store sells a limited number of 20–gallon aquariums. At the end of each week, the store manager takes inventory and places orders. Store policy is to order three new 20–gallon aquariums at the end of the week if all of the current inventory has been sold. If even one of the 20–gallon aquarium remains in stock, no new units are ordered. This policy is based on the observation that the store only sells an average of one of the 20–gallon aquariums per week. Is this policy adequate to guard against potential lost sales of 20–gallon aquariums due to a customer requesting one when they are out of stock?

waiting time between customers is exponential with $\lambda = 1$.





Transition diagram by identifying that what happens each week is governed by the same process

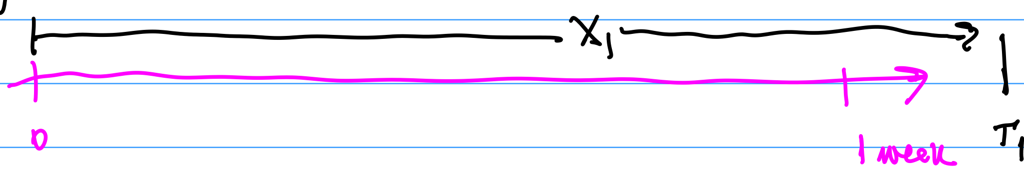


<u>Define</u> demand D as the number of customers who tried to buy an aquarium during the week.

$$P(D=0) = P(T_1>1) = P(X_1>1)$$

Nothing sold...

$\lambda = 1$



$$f(x) = \lambda e^{-\lambda x}$$

$$P(D=0) = P(X_1>1) = \int_{x>1} f(x)\,dx = \int_1^\infty \lambda e^{-\lambda x}\,dx$$

$$= \int_1^\infty e^{-x}\,dx = -e^{-x}\Big|_1^\infty = e^{-1} \approx$$

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julia> exp(-1.0)
0.36787944117144233
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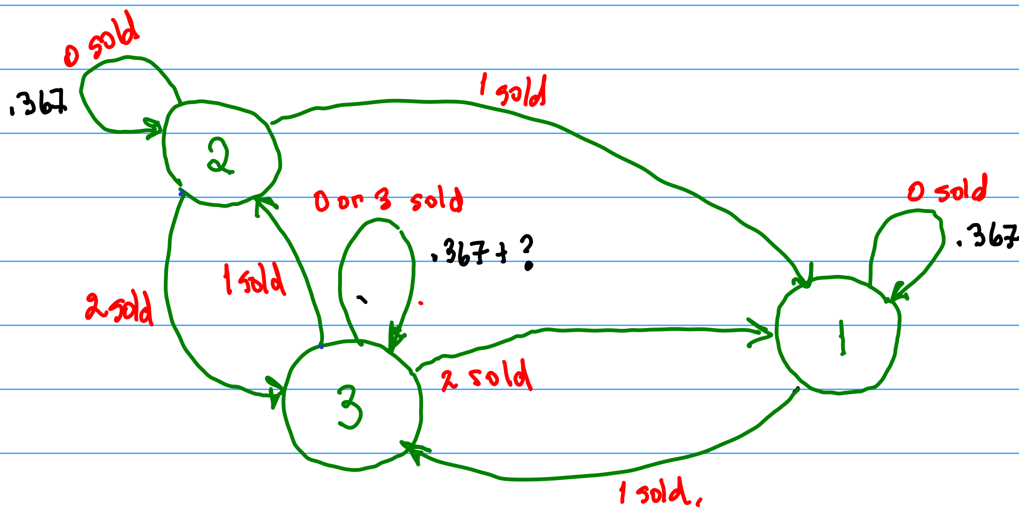
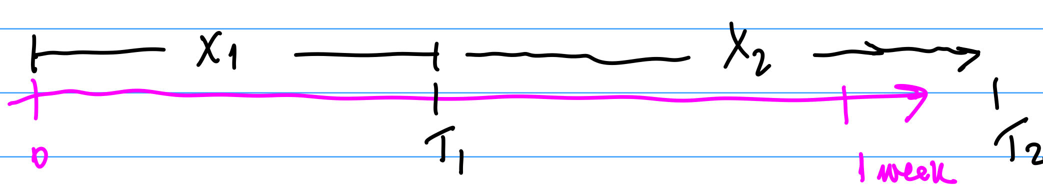


1 customer, that is $D=1$



$$P(D=1) = P(T_1 \le 1 \text{ and } T_2>1) = P(X_1 \le 1 \text{ and } X_1+X_2>1)$$

$$= P(X_1 + X_2 > 1 \text{ but not } X_1 > 1)$$

$$= \underbrace{P(X_1 + X_2 > 1)}_{\text{calculate this..}} - \underbrace{P(X_1 > 1)}_{.367}$$

$$P(X_1 + X_2 > 1) = \int_1^\infty f_{X_1+X_2}(x)\,dx$$

probability density of $X_1 + X_2$.

$$f_1(x) = \lambda e^{-\lambda x} \qquad f_2(x) = \lambda e^{-\lambda x}$$

theorem...

$$f_{X_1+X_2}(t) = (f_1 * f_2)(t) = \int_0^t f_1(t-x) f_2(x)\,dx$$

$$= \int_0^t \lambda e^{-\lambda(t-\cancel{x})} \lambda e^{\cancel{-\lambda x}}\,dx = \int_0^t \lambda e^{-\lambda t} \lambda\,dx$$

$$= \lambda^2 e^{-\lambda t} \int_0^t dx = \lambda^2 t\, e^{-\lambda t}$$

$$P(X_1 + X_2 > 1) = \int_1^\infty \lambda^2 x\, e^{-\lambda x}\,dx = \int_1^\infty x e^{-x}\,dx$$

$$u = x \qquad dv = e^{-x}\,dx$$

$$du = dx \qquad v = -e^{-x}$$

$$= x(-e^{-x})\Big|_1^\infty + \int_1^\infty e^{-x}\,dx$$

$$= e^{-1} + e^{-1} = 2e^{-1}$$

$$P(D=1) = P(X_1 + X_2 > 1) - P(X_1 > 1) = 2e^{-1} - e^{-1} = e^{-1}$$

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julia> exp(-1.0)
0.36787944117144233
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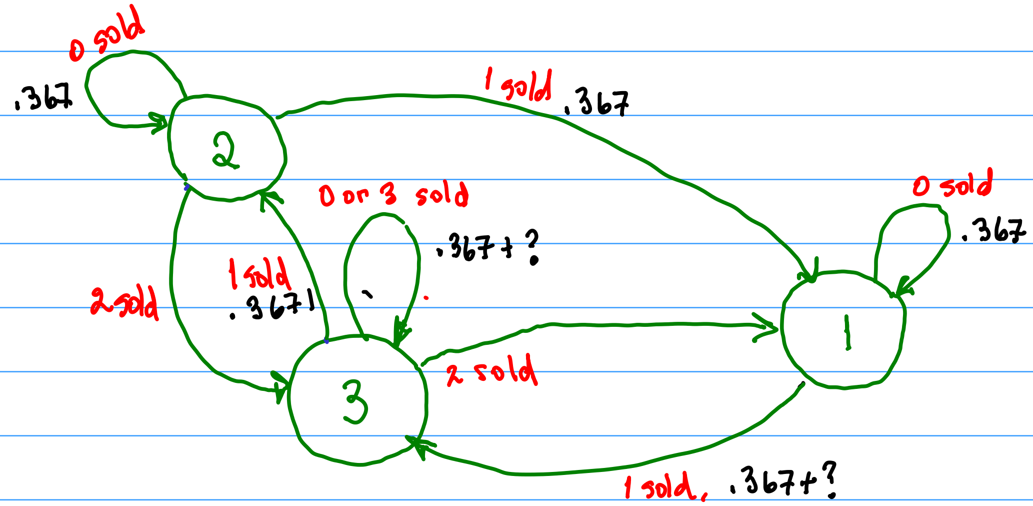

Finish filling in the probabilities next week...