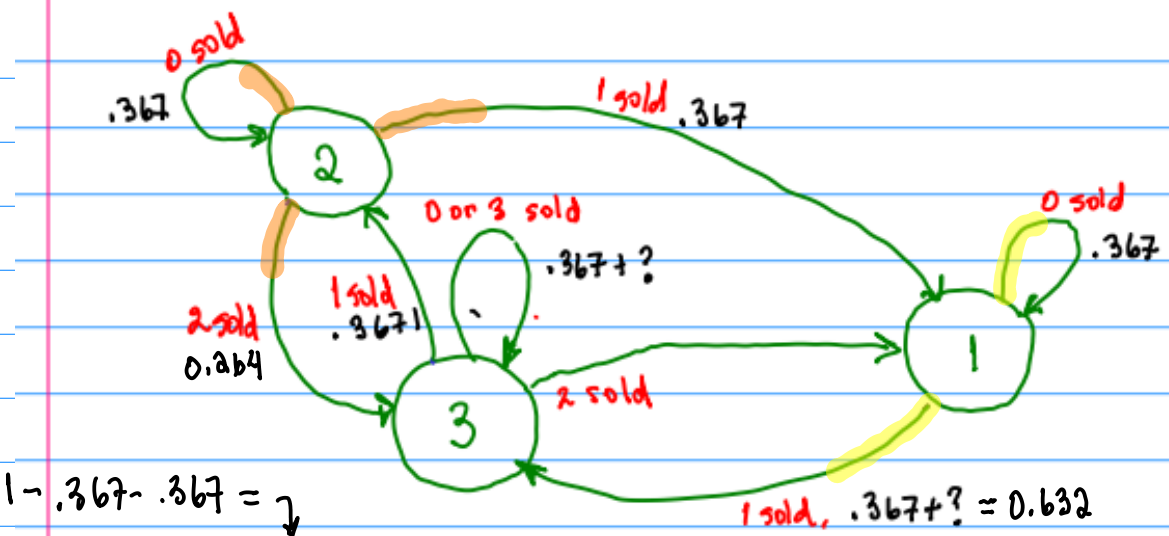


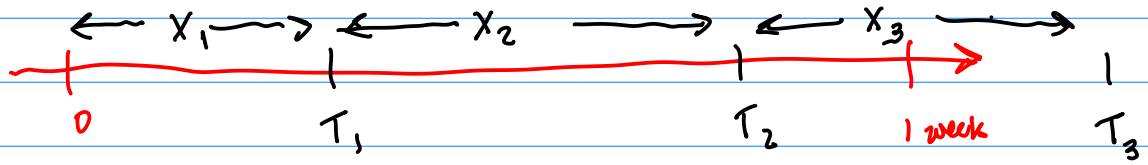
sum of the prob. of paths leading out from a state must sum to 1.

$1 - .367 - .367 =$

```
julia> 1-2*exp(-1)
0.26424111765711533
```

$1 - .367 =$

```
julia> 1-exp(-1)
0.6321205588285577
```



$$P(D=2) = P(T_2 \le 1 \text{ and } T_3 > 1)$$

$$= P(X_1 + X_2 \le 1 \text{ and } X_1 + X_2 + X_3 > 1)$$

$$= P(X_1 + X_2 + X_3 > 1 \text{ but not } X_1 + X_2 > 1)$$

$$= P(X_1 + X_2 + X_3 > 1) - P(X_1 + X_2 > 1)$$

already computed = .367

$$P(X_1 + X_2 + X_3 > 1) = \int_1^\infty f_{X_1+X_2+X_3}(x)\,dx$$

density of the variable $T_3 = X_1 + X_2 + X_3$

$$f_{X_1}(t) = \lambda e^{-\lambda t}$$

$$f_{X_1+X_2}(t) = \lambda^2 t e^{-\lambda t}$$

$$f_{X_3}(t) = \lambda e^{-\lambda t}$$

$$f_{X_1+X_2+X_3}(t) = (f_{X_1+X_2} * f_{X_3})(t) = \int_0^t f_{X_1+X_2}(s)\, f_{X_3}(t-s)\, ds$$

$$= \int_0^t \lambda^2 s e^{-\lambda s} \lambda e^{-\lambda(t-s)} ds$$

$$= \lambda^3 e^{-\lambda t} \int_0^t s\, ds = \lambda^3 e^{-\lambda t} \cdot \frac{t^2}{2} = \frac{\lambda^3 t^2}{2} e^{-\lambda t}$$

$$P(X_1+X_2+X_3 > 1) = \int_1^\infty f_{X_1+X_2+X_3}(x)\, dx = \int_1^\infty \frac{\lambda^3 x^2}{2} e^{-\lambda x} dx$$

Note $\lambda = 1$

$$= \frac{1}{2}\int_1^\infty x^2 e^{-x} dx = \frac{1}{2}\left(-x^2 e^{-x} \Big|_1^\infty + 2\int_1^\infty x e^{-x} dx \right)$$

$2e^{-1} = P(X_1+X_2>1)$

$u = x^2$ $\quad dv = e^{-x} dx$

$du = 2x\, dx$ $\quad v = -e^{-x}$

$$= \frac{1}{2} e^{-1} + P(X_1+X_2>1)$$

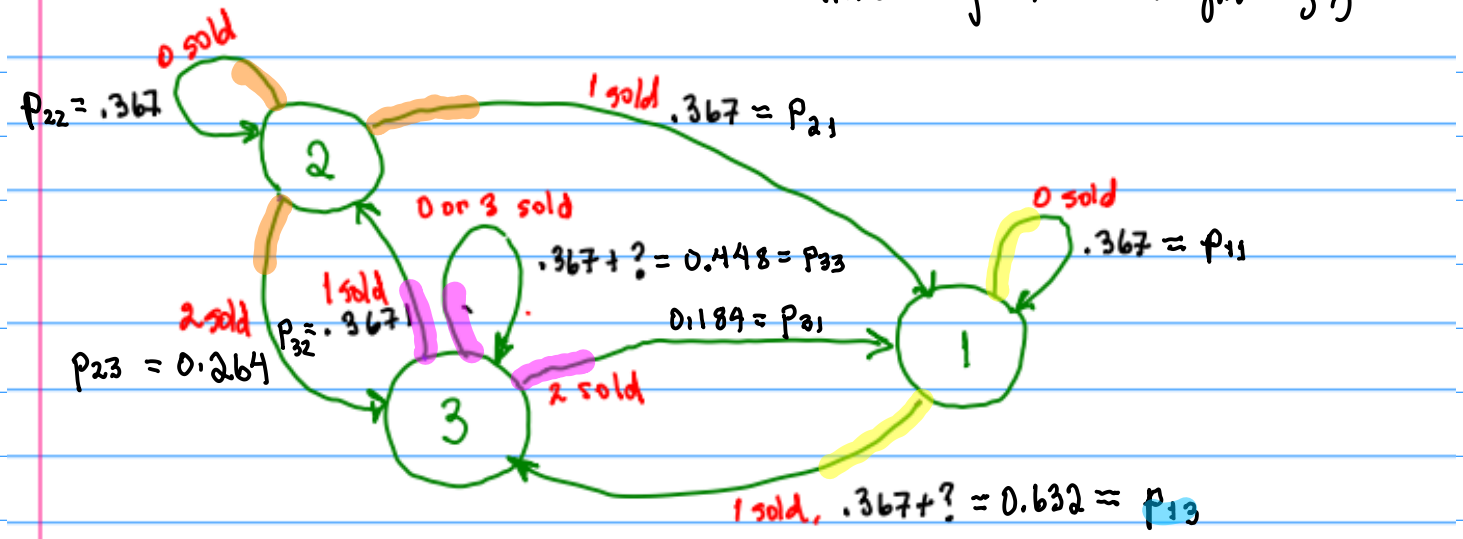
$$P(D=2) = P(X_1+X_2+X_3>1) - P(X_1+X_2>1)$$

$$= \frac{1}{2} e^{-1} + P(X_1+X_2>1) - P(X_1+X_2>1) = \downarrow$$

```
julia> 1/2*exp(-1)
0.18393972058572117
```

Transition diagram

Let S_n be the state of the inventory at the beginning of week n.



$P(D = 2) \approx 0.184$

$1 - 0.189 - 0.367 =$

```
julia> 1-(1+1/2)*exp(-1)
0.4481808382428365
```

Transition matrix

$$P(S_{n+1} = j \mid S_n = i) = p_{ij}$$

end at column index

$$P = \begin{matrix} \\ \text{start at row index} \end{matrix} \begin{bmatrix} .367 & 0 & .632 \\ .367 & .367 & .264 \\ .184 & .363 & .448 \end{bmatrix}$$

Goal is to find the expected number of aquariums in inventory and the probability there was 1, 2 or 3 aquariums...

Next time