$\lambda = 4.5$ and $\mu = 7.3$, r

$$A = \begin{array}{c} 1 \\ \\ \\ \\ \\ \end{array} \begin{bmatrix} -\lambda & \lambda & & & 0 \\ \mu & -\mu-\lambda & \ddots & & \\ & \mu & \ddots & \lambda & \\ & & \ddots & -\mu-\lambda & \lambda \\ 0 & & & \mu & -\mu \end{bmatrix} \in \mathbb{R}^{28\times 28}$$

$a_3 = [\lambda, \lambda, \ldots, \lambda] \in \mathbb{R}^{27}$

$a_2 = [-\lambda, -\mu-\lambda, \ldots, -\mu-\lambda, -\mu] \in \mathbb{R}^{28}$

$a_1 = [\mu, \mu, \ldots, \mu] \in \mathbb{R}^{27}$

$$\frac{dP}{dt} = PA$$ where $P \in \mathbb{R}^{1\times 28}$ is a row vector

note $P^T \in \mathbb{R}^{28}$ is a usual column vector

$$\frac{dP^T}{dt} = (PA)^T = A^T P^T$$

$$\frac{dY}{dt} = BY$$ where $Y = P^T$, $B = A^T$

How to solve: Find the eigenvectors and eigenvalues of B.

Let K_i be eigenvectors and β_i the corresponding eigenvalues...

$$BK_i = \beta_i K_i \quad \text{for all } i$$

General solution to $\frac{dY}{dt} = BY$ is

$$Y(t) = c_1 K_1 e^{\beta_1 t} + c_2 K_2 e^{\beta_2 t} + \cdots + c_{28} K_{28} e^{\beta_{28} t}$$

Find eigenvectors and eigenvalues by computer...

```
eigvals(A')
```

```
28-element Vector{Float64}:
 -23.190907067935594
 -22.97558296560806
 -22.61971942326365
 -22.127791631424344
 -21.505985867495507
 -20.762121699738803
 -19.90555365166377
 -18.947053563467282
 -17.8986751298931
 -16.77360231802679
 -15.58598357125999
 -14.350753884405758
 -13.0834469874733
   ⋮
  -8.014016428740039
  -6.8263976819732415
  -5.701324870106908
  -4.652946436532733
  -3.6944463483362258
  -2.837878300261149
  -2.094014132504443
  -1.4722083685756537
  -0.9802805767363099
  -0.6244170343919467
  -0.4090929320643733
  -7.277919910922247e-16
```

all β_i's are negative except

≈ 0 except for roundoff error...

$$P(t) = c_1 K_1^T e^{\beta_1 t} + c_2 K_2^T e^{\beta_2 t} + \cdots + c_{28} K_{28}^T e^{\beta_{28} t}$$

$$\lim_{t\to\infty} P(t) = 0 + 0 + \cdots + \boxed{c_{28} K_{28}^T} \cancel{e^0}^{\,1}$$

property of the exact A

The assumption of egodicity implies only one eigenvalue equal 0 and the rest negative.

average probability distribution over $[0,\infty)$.

$$\boxed{\lim_{T\to\infty} \frac{1}{T}\int_0^T P(t)\,dt} = c_{28} K_{28}^T$$

probability distribution.

$$\sum_{i=1}^{28} c_{28}(K_{28})_i = 1 \quad \text{and} \quad c_{28}(K_{28})_i \geq 0$$

components of the eigenvector

solve for the c_{28} using these equations...

$\approx K_{28}$ — the limit distribution

```
[5]: K28=eigvecs(A')[:,28]
```

```
[5]: 28-element Vector{Float64}:
 0.7874031705869717
 0.4853855161152557
 0.29921024966008897
 0.18444467444800003
 0.1136987719199999
 0.07008828406027452
 0.043205106612497934
 0.02663328489811484
 0.016417778361851525
 0.010120548305251028
 0.006238694160771468
 0.0038457703730780442
 0.002370680366965653
 ⋮
 0.00034231948309000 2
 0.0002110188594388131
 0.00013008011883233636
 8.018637462276119e-5
 4.9429956959409885e-5
 3.0470521413098395e-5
 1.8783198131294333e-5
 1.1578683779713957e-5
 7.1375447956952334e-6
 4.39985638096156e-6
 2.7122402345887854e-6
 1.6719289118143017e-6
```

```
[6]: Pe=K28/sum(K28)
```
$\approx P(\infty)$

```
[6]: 28-element Vector{Float64}:
 0.3835621458845425
 0.23644241869595048
 0.14575217590846254
 0.08984723172439471
 0.055385279830106264
 0.03414161085417539
 0.021046198471751928
 0.01297368398943596
 0.007997476431844043
 0.0049299512251093835
 0.0030390110291771567
 0.0018733629631912726
 0.0011548127855287378
 ⋮
 0.0001667516723538337
 0.00010279212679334747
 6.336500966722519e-5
 3.906062239764748e-5
 2.4078465861648048e-5
 1.4842889914599362e-5
 9.149726659651564e-6
 5.6402424615016065e-6
 3.476861791327425e-6
 2.143270967282712e-6
 1.3211944317645632e-6
 8.144349237301792e-7
```

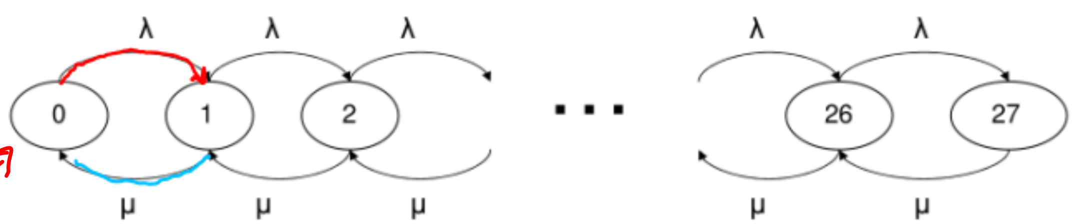
by a miracle all terms are positive...

```
[17]: sum(Pe)
[17]: 0.9999999999999999
```

sums to 1 up to rounding

answer to part (i)

(ii) Compute percentage of time the mechanic does not have any work (over a long term average).



$$\lim_{T\to 0} \frac{1}{T}\int_0^T \Pr(X_t=0)\,dt = P_0(\infty)$$

since by definition, $P_n(t) = \mathbb{P}(X_t = n)$

$\approx$

```
[8]: Pe[1]
[8]: 0.3835621458845425
[9]: 1-Pe[1]
[9]: 0.6164378541154575
```

So 38% of time no work...
pg 269 time working

$\Pr\{X_t > 0\} = 1 - P_0 = \rho \approx 0.616.$

The discrepancy in class was because I had the wrong value of λ on the computer