

Example 6.1. Two forces, which we will call red (R) and blue (B), are engaged in battle. In this conventional battle, attrition is due to direct fire (infantry) and area fire (artillery). The attrition rate due to direct fire is assumed proportional to the number of enemy infantry. The attrition rate due to artillery depends on both the amount of enemy artillery and the density of friendly troops. Red has amassed five divisions to attack a blue force of two divisions. Blue has the advantage of defense, and superior weapon effectiveness besides. How much more effective does blue have to be in order to prevail in battle?

```
In [1]: lambda=1.0
a2=0.05
b2=0.005
a1=lambda*a2
b1=lambda*b2
```

```
Out[1]: 0.005
```

```
In [2]: R0=5.0
B0=2.0
```

```
Out[2]: 2.0
```

```
In [3]: dR(R,B)=-a1*B-b1*R*B
dB(R,B)=-a2*R-b2*R*B
```

```
Out[3]: dB (generic function with 1 method)
```

```
In [4]: Rn=R0
Bn=B0
for n=2:10
    Rn=Rn+dR(Rn,Bn)
    Bn=Bn+dB(Rn,Bn)
    println("n=$n R=$Rn B=$Bn")
end
```

```
n=2 R=4.85 B=1.709
n=3 R=4.7231067499999995 B=1.43248571532125
n=4 R=4.617653549477375 B=1.1685294241077373
n=5 R=4.53224775805749 B=0.9154367118916803
n=6 R=4.465730992537333 B=0.6717096917848079
n=7 R=4.417147134005137 B=0.43601713238479733
n=8 R=4.385716518252444 B=0.20717006877346977
n=9 R=4.370815068850234 B=-0.015898194961091072
n=10 R=4.371957418948806 B=-0.23414853475149122
```

Good to visualize what is happening when modeling a problem

```
In [5]: Rn=R0; Rs=[R0]
Bn=B0; Bs=[B0]
for n=2:10
    Rn=Rn+dR(Rn,Bn); push!(Rs,Rn)
```

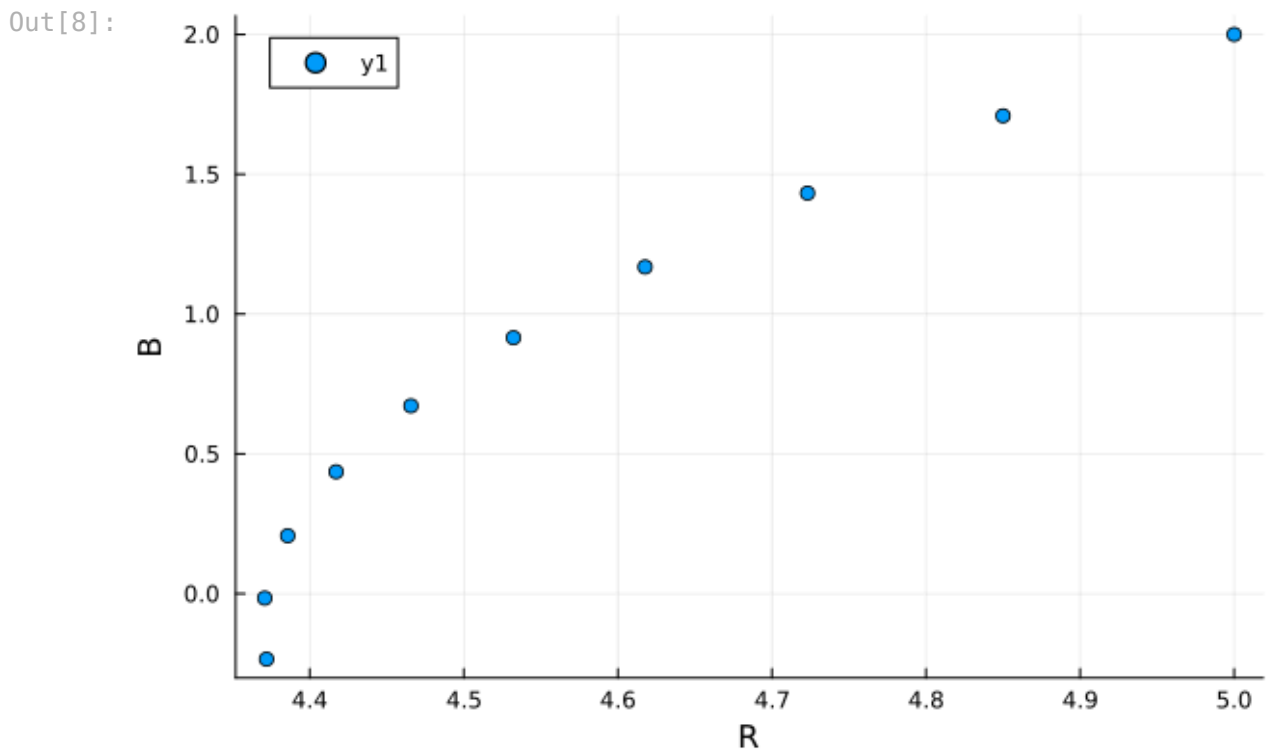
```
Bn=Bn+dB(Rn,Bn); push!(Bs,Bn)
end
```

In [6]: [Rs Bs]

```
Out[6]: 10×2 Matrix{Float64}:
 5.0      2.0
 4.85     1.709
 4.72311  1.43249
 4.61765  1.16853
 4.53225  0.915437
 4.46573  0.67171
 4.41715  0.436017
 4.38572  0.20717
 4.37082 -0.0158982
 4.37196 -0.234149
```

In [7]: **using** Plots

In [8]: scatter(Rs,Bs,xlabel="R",ylabel="B")

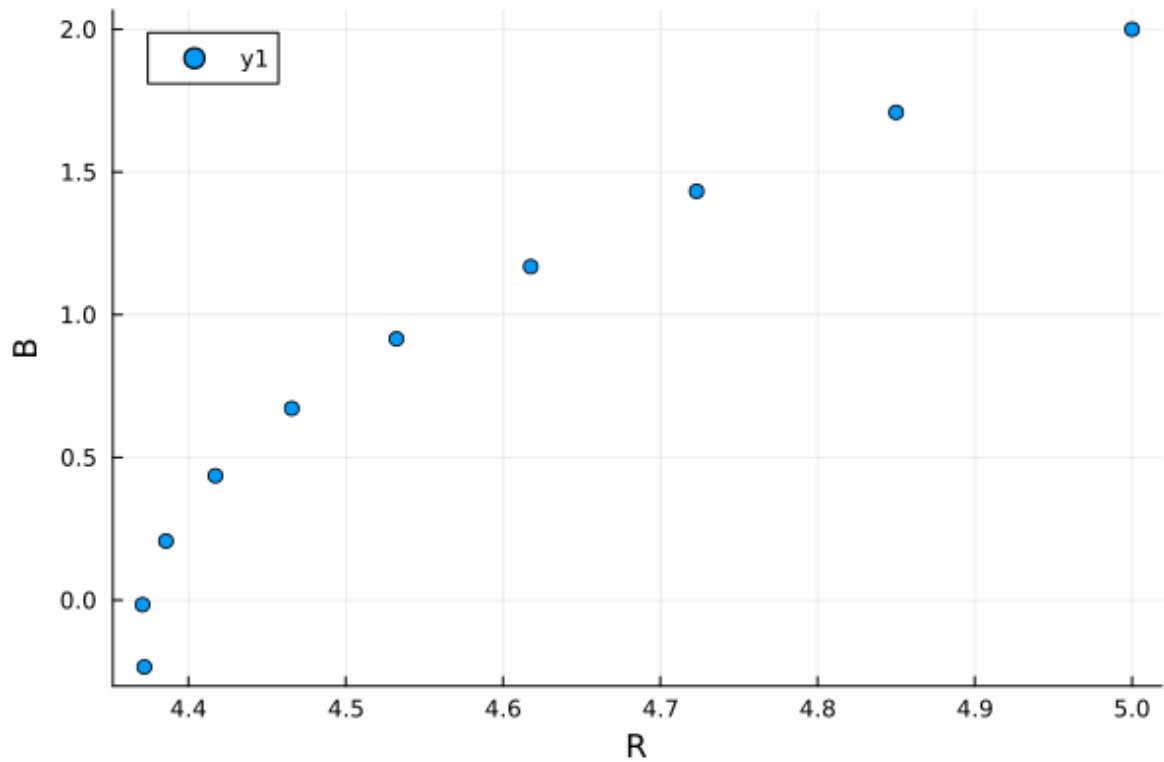


```
In [9]: function battle(lambda,nmax)
  global a1=lambda*a2
  global b1=lambda*b2
  Rn=R0; Rs=[R0]
  Bn=B0; Bs=[B0]
  for n=2:nmax
    Rn=Rn+dR(Rn,Bn); push!(Rs,Rn)
    Bn=Bn+dB(Rn,Bn); push!(Bs,Bn)
  end
```

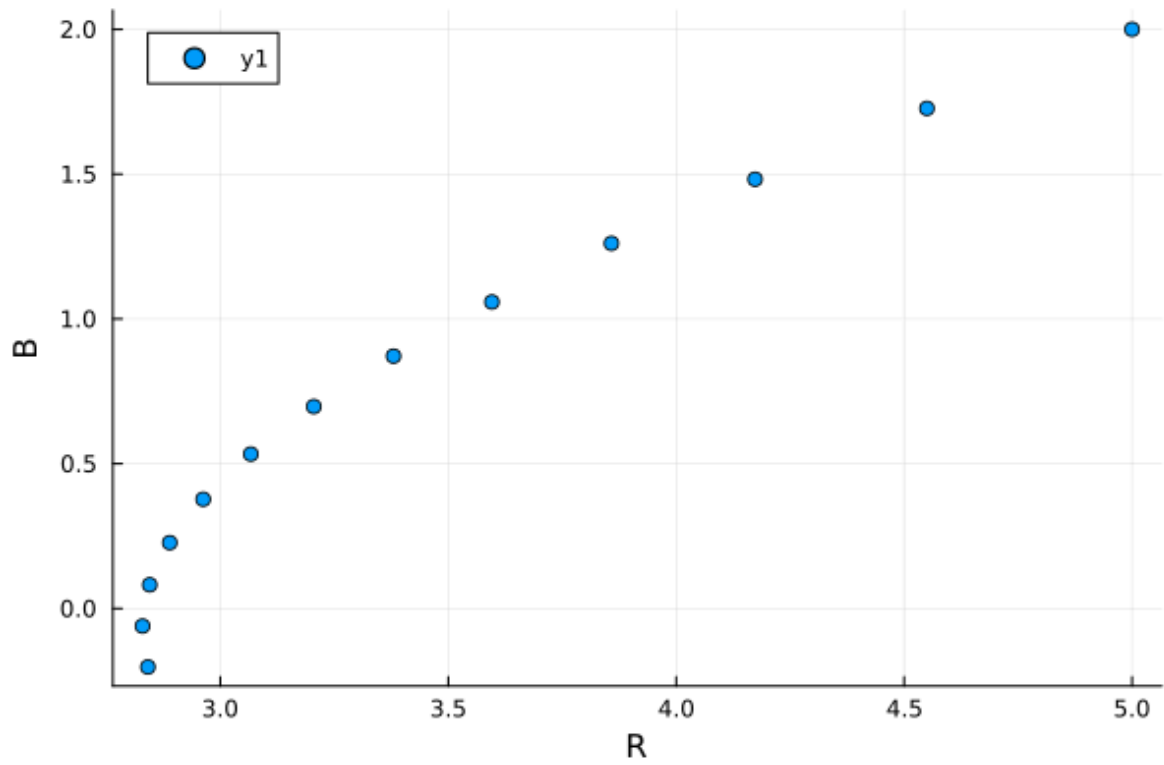
```
display(scatter(Rs,Bs,xlabel="R",ylabel="B"))  
end
```

Out[9]: battle (generic function with 1 method)

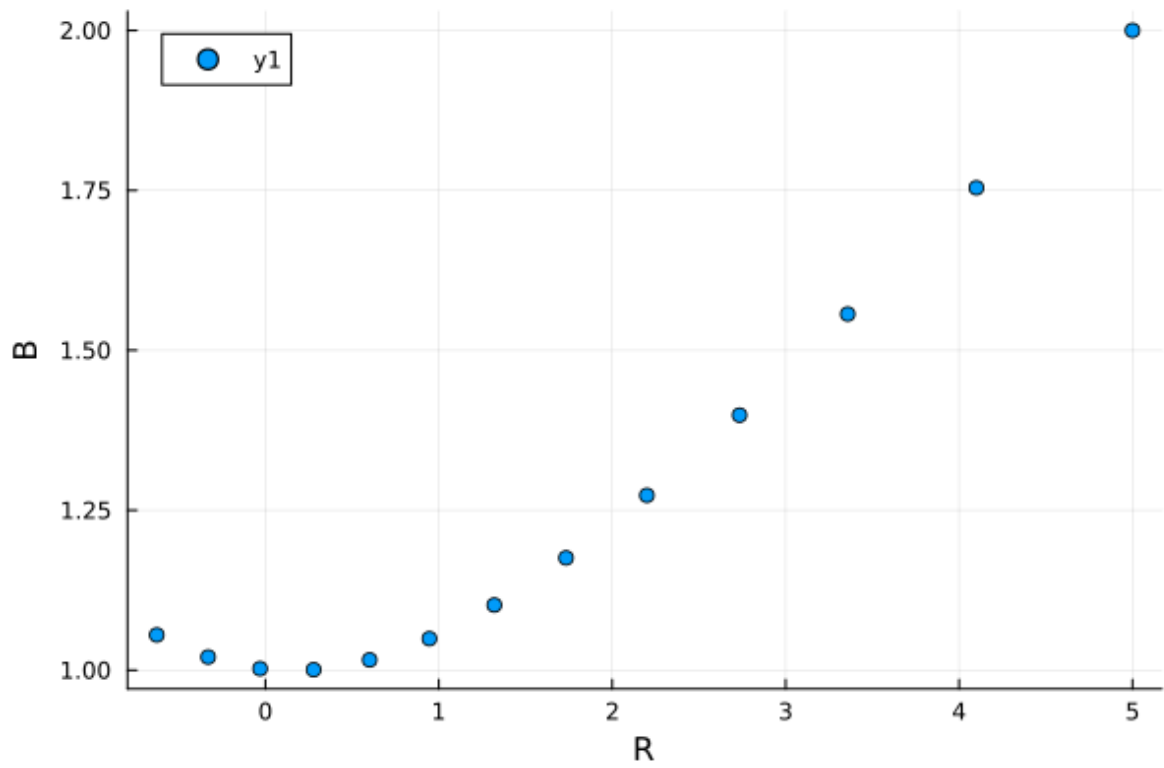
```
In [10]: battle(1.0,10)
```



```
In [11]: battle(3.0,13)
```



```
In [12]: battle(6.0,13)
```



```
In [ ]:
```