Example 7.1. An electronics manufacturer produces a variety of diodes. Quality control engineers attempt to insure that faulty diodes will be detected in the factory before they are shipped. It is estimated that 0.3% of the diodes produced will be faulty. It is possible to test each diode individually. It is also possible to place a number of diodes in series and test the entire group. If this test fails, it means that one or more of the diodes in that group are faulty. The estimated testing cost is 5 cents for a single diode, and $4+n$ cents for a group of $n>1$ diodes. If a group test fails, then each diode in the group must be retested individually to find the bad one(s). Find the most cost–effective quality control procedure for detecting bad diodes.

Let A be the expected average cost per diode

In [1]:
```
A(n)=4/n+6-5*p^n
```

Out[1]:
```
A (generic function with 1 method)
```

In [2]:
```
q=0.003
p=1-q
```

Out[2]:
```
0.997
```

In [3]:
```
T=[2:20 A.(2:20)]
```

Out[3]:
```
19×2 Matrix{Float64}:
  2.0  3.02996
  3.0  2.3782
  4.0  2.05973
  5.0  1.87455
  6.0  1.75599
  7.0  1.67549
  8.0  1.61875
  9.0  1.57784
 10.0  1.54799
 11.0  1.52618
 12.0  1.51039
 13.0  1.49922
 14.0  1.49167
 15.0  1.487
 16.0  1.48467
 17.0  1.48426
 18.0  1.48545
 19.0  1.48796
 20.0  1.4916
```

In [4]:
```
i=argmin(A.(2:20))
```

Out[4]:
```
16
```

In [5]:
```
T[i,:]
```

Out[5]:
```
2-element Vector{Float64}:
 17.0
  1.484264961220581
```

The optimial batch size is 17 and the per-diode testing cost is 1.484264961220581

In [6]:
```
nopt=Int(T[i,1])
```

Out[6]:
```
17
```

In [7]:
```
Aopt=T[i,2]
```

Out[7]:
```
1.484264961220581
```

In [8]:
```
using Symbolics
D(f,x)=expand_derivatives(Differential(x)(f))
@variables q,p,n
```

Out[8]:
```
3-element Vector{Num}:
 q
 p
 n
```

In [9]:
```
A(n)
```

Out[9]:
```
6 - 5(p^n) + 4 / n
```

In [10]:
```
Anq=substitute(A(n),p=>1-q)
```

Out[10]:
```
6 - 5((1 - q)^n) + 4 / n
```

In [11]:
```
dAdq=D(Anq,q)
```

Out[11]:
```
5n*((1 - q)^(-1 + n))
```

In [12]:
```
SAq=substitute(q/Aopt*dAdq,[n=>nopt,q=>0.003])
```

Out[12]:
```
0.16373867744143392
```

The relative sensitivity $S(A,q) = 0.16373867744143392$

In [ ]:
```

```