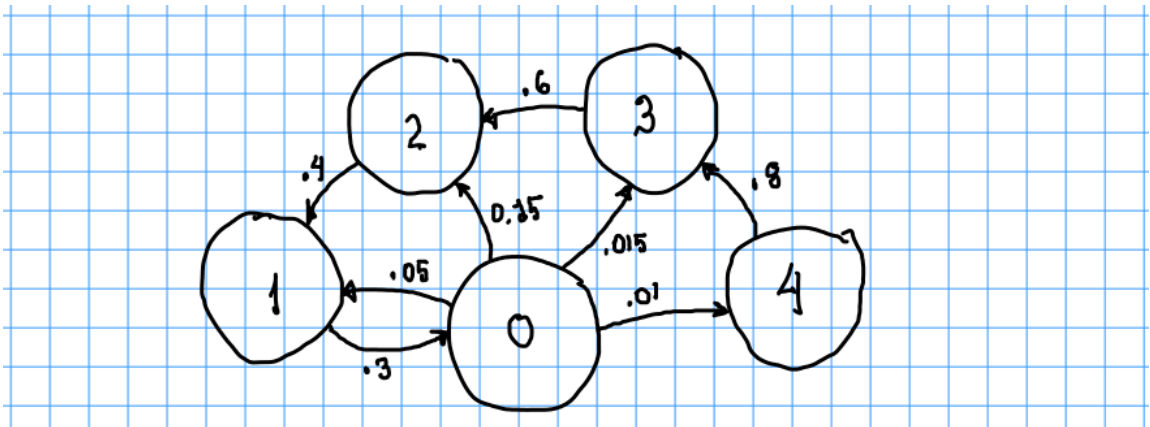


Math 420/620 Final Question 3

A Markov chain model of floods uses the state variable $X_n = 0, 1, 2, 3, 4$ where state 0 means the average daily flow is below 1000 cubic feet per second, state 1 is 1000-2000 cfs, state 2 is 2000-5000 cfs, 3 is 5000-10000 cfs and 4 is over 10000 cfs. The state transition probability matrix for this model is

$$P = \begin{bmatrix} 0.9 & 0.05 & 0.025 & 0.015 & 0.01 \\ 0.3 & 0.7 & 0 & 0 & 0 \\ 0 & 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 & 0 \\ 0 & 0 & 0 & 0.8 & 0.2 \end{bmatrix} .$$

(i) Draw the state transition probability diagram for this model.



(ii) Find the steady-state probability distribution for this model.

```
In [1]: P=[0.9 0.05 0.025 0.015 0.01;
          0.3 0.7 0 0 0;
          0 0.4 0.6 0 0;
          0 0 0.6 0.4 0;
          0 0 0 0.8 0.2]
```

```
Out[1]: 5x5 Matrix{Float64}:
 0.9  0.05  0.025  0.015  0.01
 0.3  0.7   0.0   0.0   0.0
 0.0  0.4   0.6   0.0   0.0
 0.0  0.0   0.6   0.4   0.0
 0.0  0.0   0.0   0.8   0.2
```

```
In [2]: using LinearAlgebra
        K5=eigvecs(P')[ :,5]
```

```
Out[2]: 5-element Vector{ComplexF64}:
  -0.941292284529958 + 0.0im
  -0.3137640948433191 + 0.0im
  -0.1176615355662446 + 0.0im
  -0.039220511855414884 + 0.0im
  -0.01176615355662446 + 0.0im
```

```
In [3]: K5=real.(K5)
println("The steady state is as follows:")
PI=K5/sum(K5)
```

The steady state is as follows:

```
Out[3]: 5-element Vector{Float64}:
  0.6611570247933886
  0.22038567493112934
  0.08264462809917347
  0.027548209366391168
  0.008264462809917347
```

(iii) How often are severe floods (over 10000 cfs) expected to occur?

Since

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \mathbf{P}\{X_n = 4\} = \pi_4 \approx 0.008264462809917349.$$

I would expect 0.8% chance each day for there to be a sever flood.

Since there are 365 days in a year that means about 3.0165 times a year.

```
In [4]: 365*PI[5]
```

```
Out[4]: 3.0165289256198315
```

(iv) A reservoir used for drought storage depends on the flow of this river. When the flow is over 5000 cfs, the reservoir is allowed to store 1000 acre-feet per day. When the flow is under 1000 cfs the reservoir is required to release 100 acre-feet per day back into the river. Find the expected annual number of acre-feet of water stored in the reservoir. Is this number positive or negative? What does this mean?

Let R_n represent water flowing in or out of the reservoir on day n . Thus,

$$R_n = \begin{cases} 1000 & \text{if } X_n \geq 3 \\ -100 & \text{if } X_n = 0 \\ 0 & \text{otherwise.} \end{cases}$$

The expected value of R_n per day is

$$\mathbf{E}[R_n] = 1000\mathbf{P}\{X_n \geq 3\} - 100\mathbf{P}\{X_n = 0\}.$$

Therefore, provided n is large

$$\mathbf{E}[R_n] \approx 1000(\pi_3 + \pi_4) - 100\pi_0.$$

```
In [5]: # All the indices are off by one
ER=1000*(PI[4]+PI[5]) - 100*PI[1]
```

```
Out[5]: -30.30303030303034
```

```
In [6]: cf=ER*365
```

```
Out[6]: -11060.606060606075
```

The annual number of acre-feet of water stored in the reservoir is negative. This means we expect to release 11060.61 acre-feet of water from the reservoir per year and the reservoir will eventually run out of water.