Idea is to look at the solutions to

$$\frac{dX}{dt} = AX$$

for different matrices A and see what happens.

In [1]:
```
using LinearAlgebra, Plots
```

Example 1 Take eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -2$ with corresponding eigenvectors $u_1 = (1, 1)$ and $u_2 = (1, -1)$.

In [2]:
```
# Specify the eigenvalues as diagonal elements
D=[1 0; 0 -2]
```

Out[2]:
```
2×2 Matrix{Int64}:
 1   0
 0  -2
```

In [3]:
```
# Specify the eigenvectors as the columns of U
# or the rows of U'
U=[1 1; 1 -1]'
```

Out[3]:
```
2×2 adjoint(::Matrix{Int64}) with eltype Int64:
 1   1
 1  -1
```

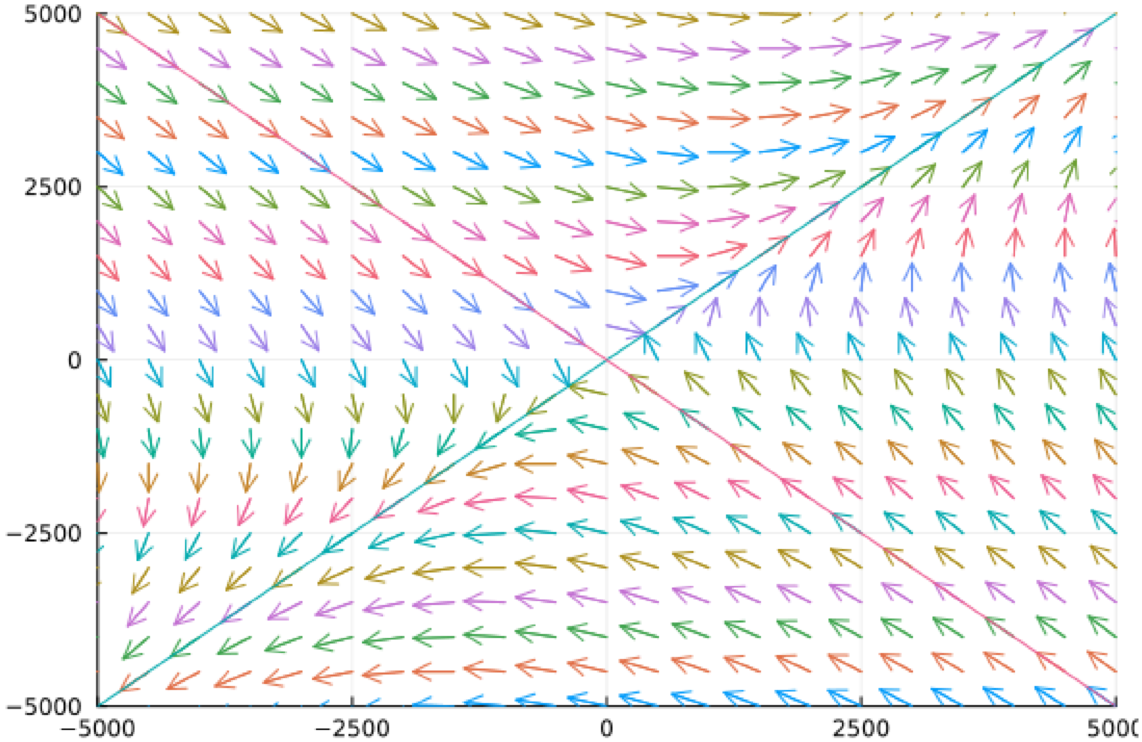
In [4]:
```
# build the matrix A with specified eigenvalues and vectors
A=U*D*inv(U)
```

Out[4]:
```
2×2 Matrix{Float64}:
 -0.5   1.5
  1.5  -0.5
```

Now draw the direction field corresponding to $dX/dt = AX$.

In [5]:
```
xs=-5000:500:5000
ys=-5000:500:5000
quiver(
 xs*ones(length(ys))',
 ones(length(xs))*ys',
 quiver=(x,y)->A*[x,y]/norm(A*[x,y])*400,
)
# add parametric curves along the eigenvectors
v1(t)=U[:,1]*t
v2(t)=U[:,2]*t
plot!(t->v1(t)[1],t->v1(t)[2],-5000:500:5000,
    xlim=(-5000,5000),ylim=(-5000,5000))
plot!(t->v2(t)[1],t->v2(t)[2],-5000:500:5000,
    xlim=(-5000,5000),ylim=(-5000,5000),legend=false)
```

Out[5]:



Example 2 Take eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -2$ with random eigenvectors u_1 and u_2 sampled from a standard normal distribution.

In [6]:
```
# Specify the exact random matrix from class (repeatability)
#U=randn(2,2)
U=[-1.3522 -0.965533; -0.425913 0.0980373]
```

Out[6]:
```
2×2 Matrix{Float64}:
 -1.3522    -0.965533
 -0.425913   0.0980373
```

In [7]:
```
# build the matrix A with specified eigenvalues and vectors
A=U*D*inv(U)
```

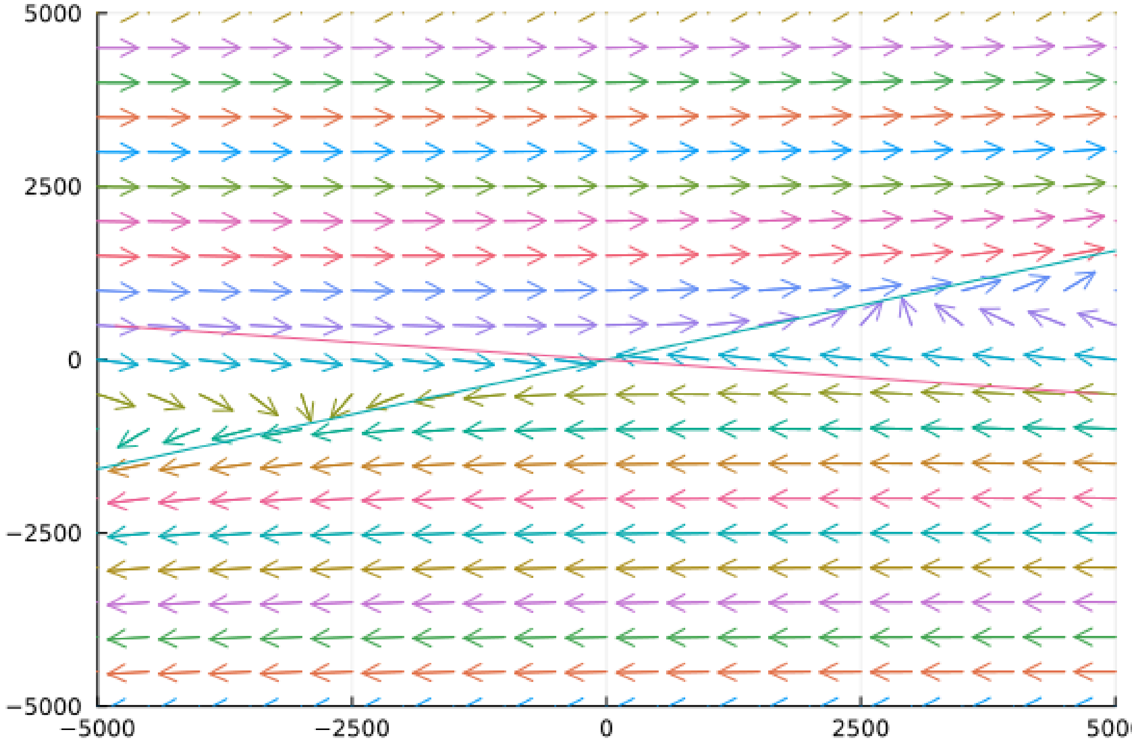
Out[7]:
```
2×2 Matrix{Float64}:
 -1.26867   7.20263
  0.230354  0.268667
```

Now draw the direction field corresponding to $dX/dt = AX$.

In [8]:
```
xs=-5000:500:5000
ys=-5000:500:5000
quiver(
 xs*ones(length(ys))',
 ones(length(xs))*ys',
 quiver=(x,y)->A*[x,y]/norm(A*[x,y])*400,
)
# add parametric curves along the eigenvectors
v1(t)=U[:,1]*t
v2(t)=U[:,2]*t
plot!(t->v1(t)[1],t->v1(t)[2],-5000:500:5000,
```

```
    xlim=(-5000,5000),ylim=(-5000,5000))
plot!(t->v2(t)[1],t->v2(t)[2],-5000:500:5000,
    xlim=(-5000,5000),ylim=(-5000,5000),legend=false)
```

Out[8]:



Example 3 Now try pure imaginary eigenvalues. So the resulting matrix $A \in \mathbf{R}^2$ we choose $\lambda_1 = \overline{\lambda_2}$ and $u_1 = \overline{u_2}$. Thus, λ_1 and λ_2 are complex conjugates of each other as are u_1 and u_2.

In [9]:

```
# Specify the eigenvalues as diagonal elements
D=[1im 0; 0 -1im]
```

Out[9]:

```
2×2 Matrix{Complex{Int64}}:
 0+1im  0+0im
 0+0im  0-1im
```

In [10]:

```
# Specify the eigenvectors as the columns of U
# or the rows of U'
U=[1 1im; 1 -1im]'
```

Out[10]:

```
2×2 adjoint(::Matrix{Complex{Int64}}) with eltype Complex{Int64}:
 1+0im  1+0im
 0-1im  0+1im
```

In [11]:

```
A=U*D*inv(U)
```

Out[11]:

```
2×2 Matrix{ComplexF64}:
 0.0+0.0im  -1.0+0.0im
 1.0+0.0im   0.0+0.0im
```

In [12]:

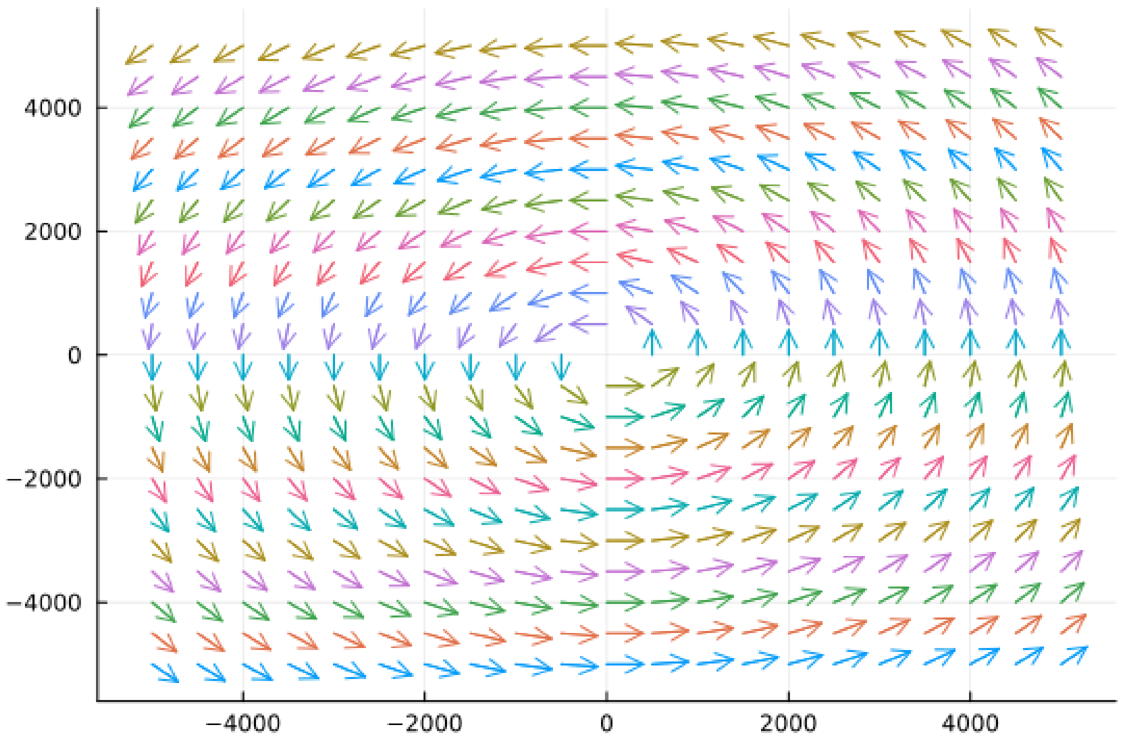
```
# Since the matrix is real, force the type for convenience
A=real.(A)
```

Out[12]:
```
2×2 Matrix{Float64}:
 0.0  -1.0
 1.0   0.0
```

Now draw the direction field corresponding to $dX/dt = AX$.

In [13]:
```
xs=-5000:500:5000
ys=-5000:500:5000
quiver(
 xs*ones(length(ys))',
 ones(length(xs))*ys',
 quiver=(x,y)->A*[x,y]/norm(A*[x,y])*400,
)
```

Out[13]:



In [ ]: