Example 6.6. Consider a layer of air that is heated from the bottom. In certain situations the warmer air rising up interacts with the colder air sinking down to form turbulent convection rolls. The complete derivation of the dynamics of motion involves a system of partial differential equations, which can be solved by the method of Fourier transforms; see Lorentz (1963). A simplified representation involves three state variables. The variable x1 represents the rate at which the convection rolls rotate, x2 represents the temperature difference between the ascending and descending air currents, and x3 represents the deviation from linearity of the vertical temperature profile, a positive value indicating that the temperature varies faster near the boundary. The equations of motion for this system are $dX/dt = F(X)$ where

$$F(X) = F(x_1, x_2, x_3) = \begin{bmatrix} -\sigma x_1 + \sigma x_2 \\ -x_2 + r x_1 - x_1 x_3 \\ -b x_3 + x_1 x_2 \end{bmatrix}$$

and we will consider the realistic case where σ = 10 and b = 8/3. The remaining parameter r represents the temperature difference between the top and bottom of the air layer. Increasing r pumps more energy into the system, creating more vigorous dynamics.

In [1]:
```
F(x1,x2,x3)=[-sigma*x1+sigma*x2,
             -x2+r*x1-x1*x3,
             -b*x3+x1*x2]
```

Out[1]:
```
F (generic function with 1 method)
```

In [2]:
```
sigma=10; b=8/3; r=8
```

Out[2]:
```
8
```

In [3]:
```
function mktraj(X0)
    h=1/64
    Xn=X0
    Xtraj=[X0]
    for n=1:10000
        Xn=Xn+h*F(Xn...)
        push!(Xtraj,Xn)
    end
    return Xtraj
end
```

Out[3]:
```
mktraj (generic function with 1 method)
```

In [5]:
```
Xtraj=mktraj([1.0,1,1]);
```

In [6]:
```
Xtraj
```

Out[6]:
```
10001-element Vector{Vector{Float64}}:
 [1.0, 1.0, 1.0]
 [1.0, 1.09375, 0.9739583333333334]
 [1.0146484375, 1.1864420572916667, 0.9504665798611112]
 [1.041491190592448, 1.2796663699878588, 0.9296735178817203]
 [1.078706062372981, 1.3747291258589653, 0.9117615158539367]
 [1.124959666042666, 1.4727196992735725, 0.8969422127274931]
 [1.1792971712349951, 1.57456241516385 8, 0.8854563433597213]
 [1.24105736559888, 1.6810561463160438, 0.8775760322114363]
 [1.3098071750859368, 1.7929042803758244, 0.8736086003334201]
 [1.3852910977874817, 1.9107370039286085, 0.8739013496530466]
 [1.467392020622032 8, 2.035127379215365, 0.8788470271949315]
 [1.5561006704022409, 2.166602311570172, 0.8888898022682165]
 [1.6514915518347302, 2.305649202733054, 0.9045316538851041]
 ⋮
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
 [4.320493798938575, 4.3204937989385765, 6.9999999999999964]
```

In [7]:
```
second(X)=X[2]
```

Out[7]:
```
second (generic function with 1 method)
```

In [8]:
```
second.(Xtraj)
```

Out[8]:
```
10001-element Vector{Float64}:
 1.0
 1.09375
 1.1864420572916667
 1.2796663699878588
 1.3747291258589653
 1.4727196992735725
 1.574562415163858
 1.6810561463160438
 1.7929042803758244
 1.9107370039286085
 2.035127379215365
 2.166602311570172
 2.305649202733054
 ⋮
 4.3204937989385765
 4.3204937989385765
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 4.3204937989385765
 4.3204937989385765
 4.3204937989385765
```
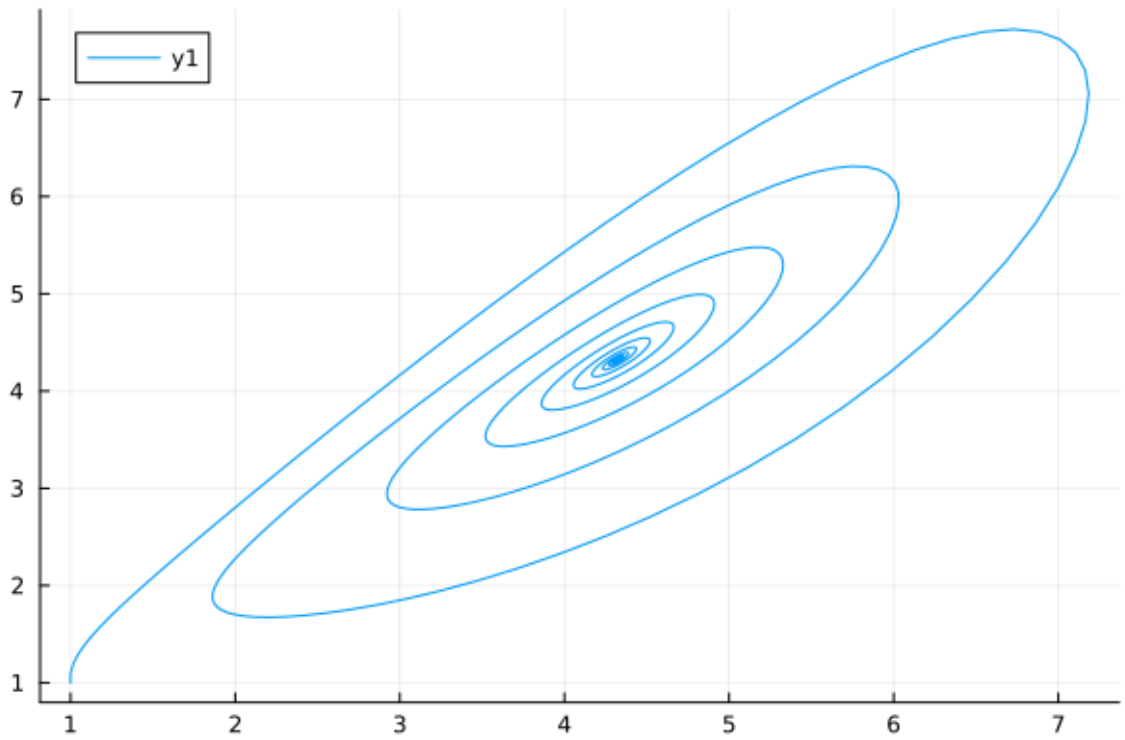

In [10]:
```
using Plots
```

In [11]:
```
plot(first.(Xtraj),second.(Xtraj))
```

Out[11]:



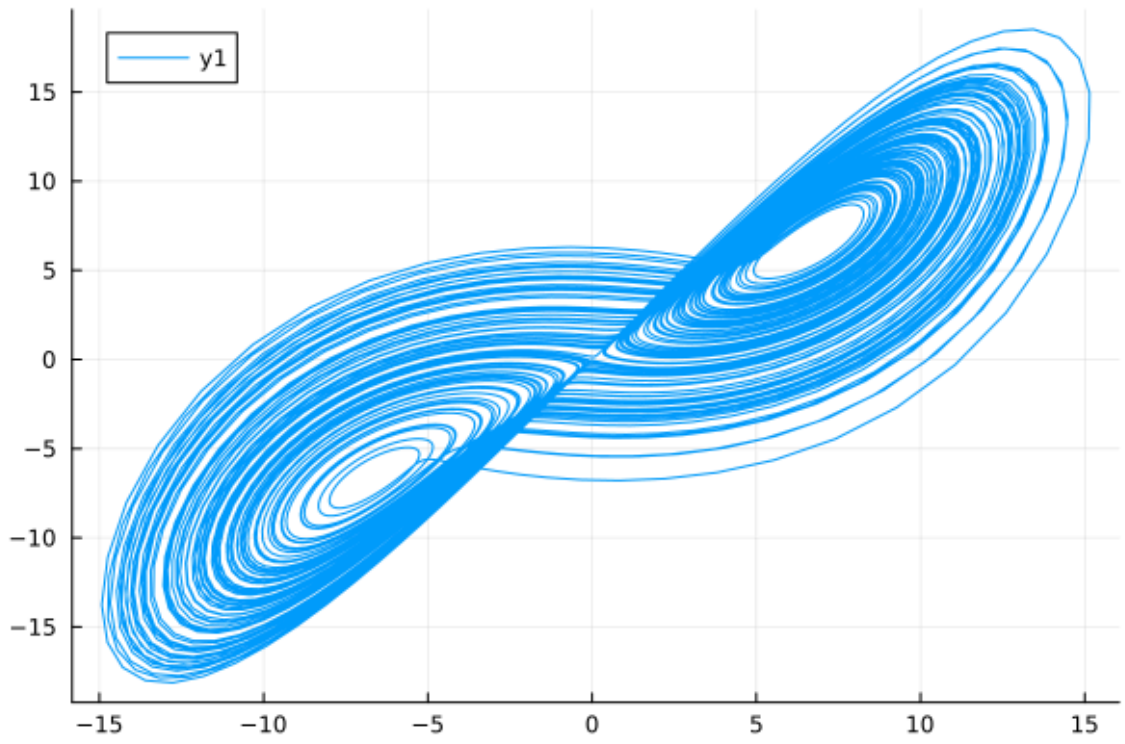
In [12]:
```
r=18
```

Out[12]: 18

In [13]:
```
Xtraj2=mktraj([1.0,1,1]);
```

In [14]:
```
plot(first.(Xtraj2),second.(Xtraj2))
```

Out[14]:



In [ ]: