

Example 6.6. Consider a layer of air that is heated from the bottom. In certain situations the warmer air rising up interacts with the colder air sinking down to form turbulent convection rolls. The complete derivation of the dynamics of motion involves a system of partial differential equations, which can be solved by the method of Fourier transforms; see Lorentz (1963). A simplified representation involves three state variables. The variable  $x_1$  represents the rate at which the convection rolls rotate,  $x_2$  represents the temperature difference between the ascending and descending air currents, and  $x_3$  represents the deviation from linearity of the vertical temperature profile, a positive value indicating that the temperature varies faster near the boundary. The equations of motion for this system are  $dX/dt = F(X)$  where

$$F(X) = F(x_1, x_2, x_3) = \begin{bmatrix} -\sigma x_1 + \sigma x_2 \\ -x_2 + r x_1 - x_1 x_3 \\ -b x_3 + x_1 x_2 \end{bmatrix}$$

and we will consider the realistic case where  $\sigma = 10$  and  $b = 8/3$ . The remaining parameter  $r$  represents the temperature difference between the top and bottom of the air layer. Increasing  $r$  pumps more energy into the system, creating more vigorous dynamics.

```
In [1]: F(x1,x2,x3)=[-sigma*x1+sigma*x2,
                    -x2+r*x1-x1*x3,
                    -b*x3+x1*x2]
```

```
Out[1]: F (generic function with 1 method)
```

```
In [2]: sigma=10; b=8/3; r=18
```

```
Out[2]: 18
```

```
In [3]: function mktraj(X0,nmax)
        h=1/64
        Xn=X0
        Xtraj=[X0]
        for n=1:nmax
            Xn=Xn+h*F(Xn...)
            push!(Xtraj,Xn)
        end
        return Xtraj
end
```

```
Out[3]: mktraj (generic function with 1 method)
```

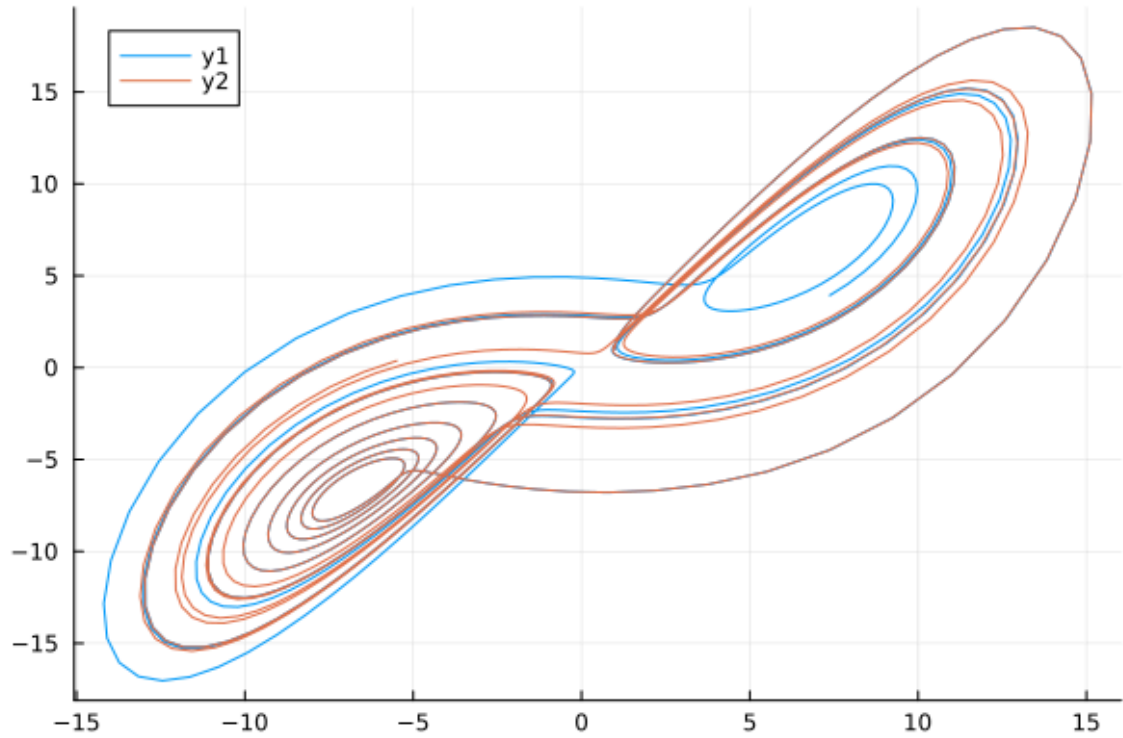
```
In [4]: second(X)=X[2]
```

```
Out[4]: second (generic function with 1 method)
```

In [5]: `using Plots`

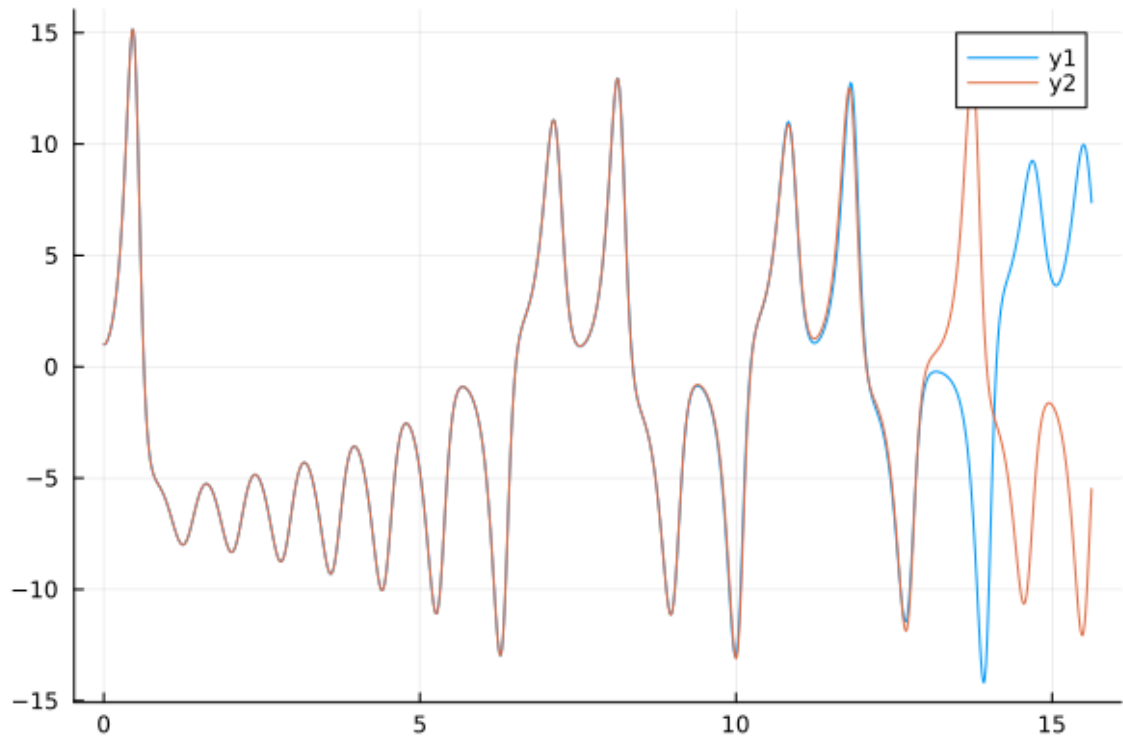
```
In [6]: Xtraj2=mktraj([1.0,1,1],1000)
Xtraj2e=mktraj([1.01,1,1],1000)
plot(first.(Xtraj2),second.(Xtraj2))
plot!(first.(Xtraj2e),second.(Xtraj2e))
```

Out[6]:



```
In [7]: plot(1/64*(0:1000),first.(Xtraj2))
plot!(1/64*(0:1000),first.(Xtraj2e))
```

Out[7]:



In [ ]: