Example 5.3. Consider the electrical circuit diagrammed in Figure 5.6. The circuit consists of a capacitor, a resistor, and an inductor in a simple closed loop. The effect of each component of the circuit is measured in terms of the relationship between current and voltage on that branch of the loop. An idealized physical model gives the relations

$$C\frac{dv_C}{dt} = i_C \qquad \text{(capacitor)}$$
$$v_R = f(i_R) \qquad \text{(resistor)}$$
$$L\frac{di_L}{dt} = v_L \qquad \text{(inductor)}$$

where v_C represents the voltage across the capacitor, i_R represents the current through the resistor, and so on. The function $f(x)$ is called the v-i characteristic of the resistor. Usually $f(x)$ has the same sign as x. This is called a passive resistor. Kirchoff's current law states that the sum of the currents flowing into a node equals the sum of the currents flowing out. Kirchoff's voltage law states that the sum of the voltage drops along a closed loop must add up to zero. Determine the behavior of this circuit over time in the case where $L = 1$, $C = 1/3$, and $f(x) = x^3 + 4x$.

The linearized equation is $dX/dt = AX$ where

$$A = \begin{bmatrix} -4 & -1 \\ 3 & 0 \end{bmatrix}.$$

In [1]:
```
A=[-4 -1; 3 0]
```

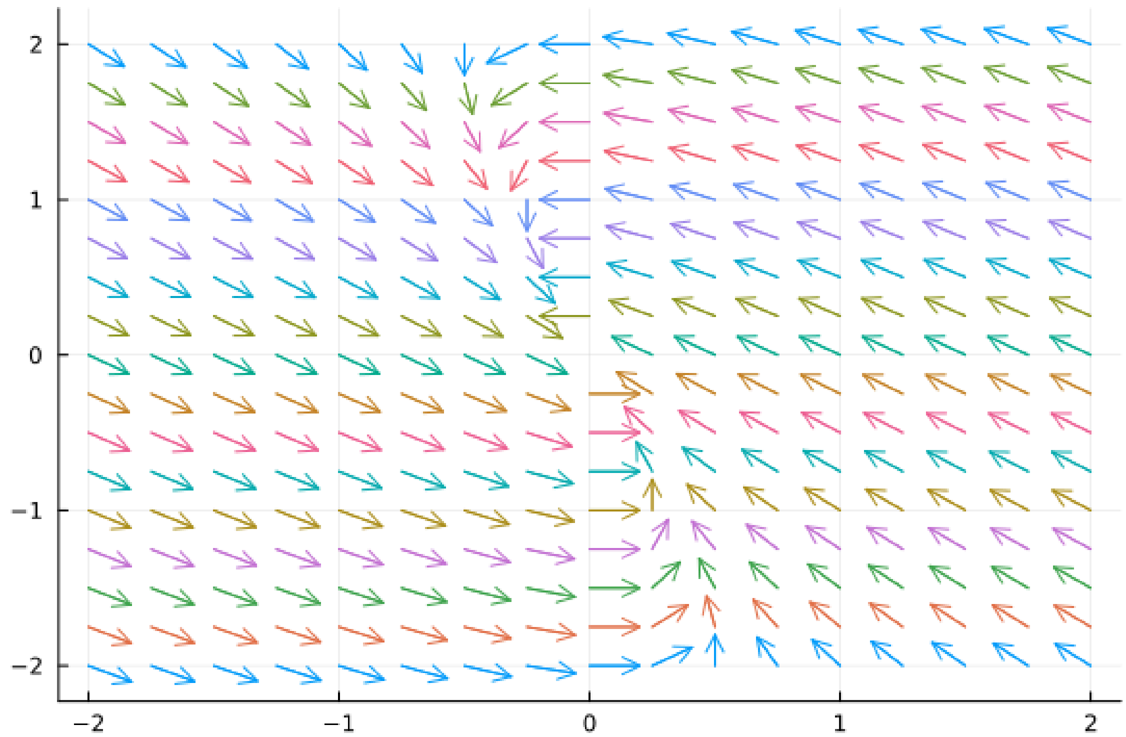
Out[1]:
```
2×2 Matrix{Int64}:
 -4  -1
  3   0
```

In [2]:
```
using Plots, LinearAlgebra
```

The direction field for the linearized equation.

In [3]:
```
xs=-2:0.25:2
ys=-2:0.25:2
quiver(
xs*ones(length(ys))',
ones(length(xs))*ys',
quiver=(x,y)->A*[x,y]/norm(A*[x,y])*0.2,
)
```

Out[3]:



Solutions to the linearized equation are

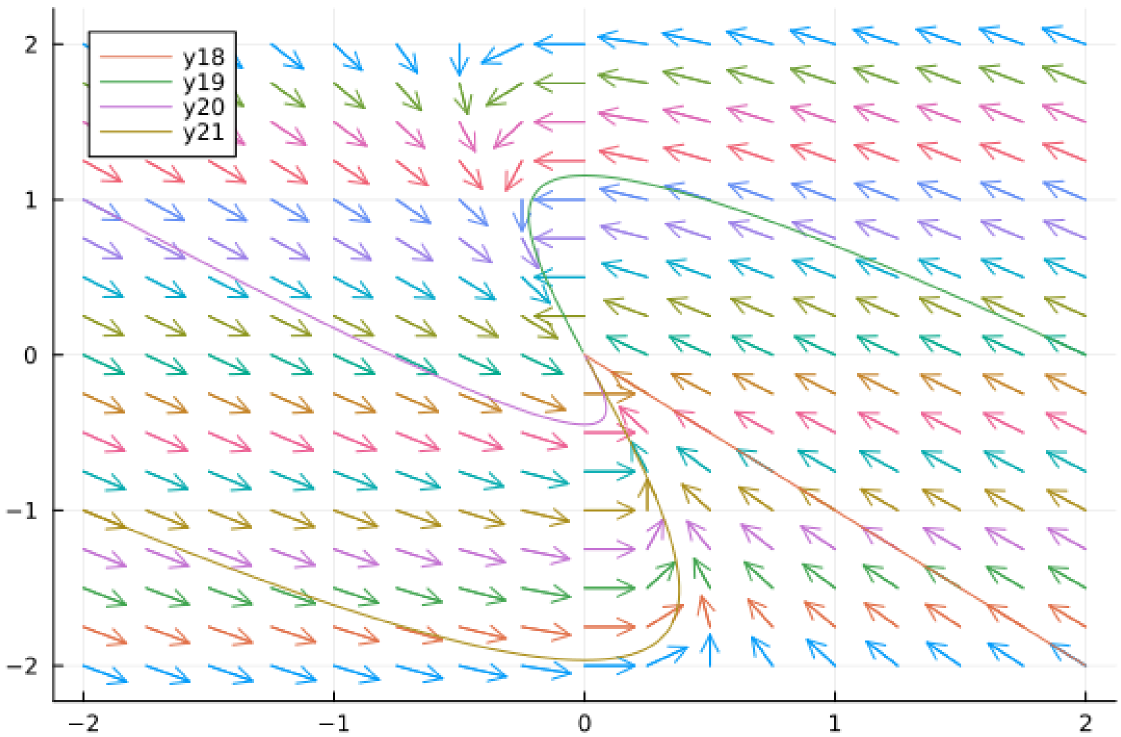
$$X(t) = e^{At}c$$

where c is the initial condition at time $t = 0$.

In [4]:

```
X(t)=exp(A*t)*[2,-2]
plot!(t->X(t)[1],t->X(t)[2],0:0.01:5)
X(t)=exp(A*t)*[2,0]
plot!(t->X(t)[1],t->X(t)[2],0:0.01:5)
X(t)=exp(A*t)*[-2,1]
plot!(t->X(t)[1],t->X(t)[2],0:0.01:5)
X(t)=exp(A*t)*[-2,-1]
plot!(t->X(t)[1],t->X(t)[2],0:0.01:5)
```

Out[4]:



In [5]:
```
eigvals(A)
```

Out[5]:
```
2-element Vector{Float64}:
 -3.0
 -1.0
```

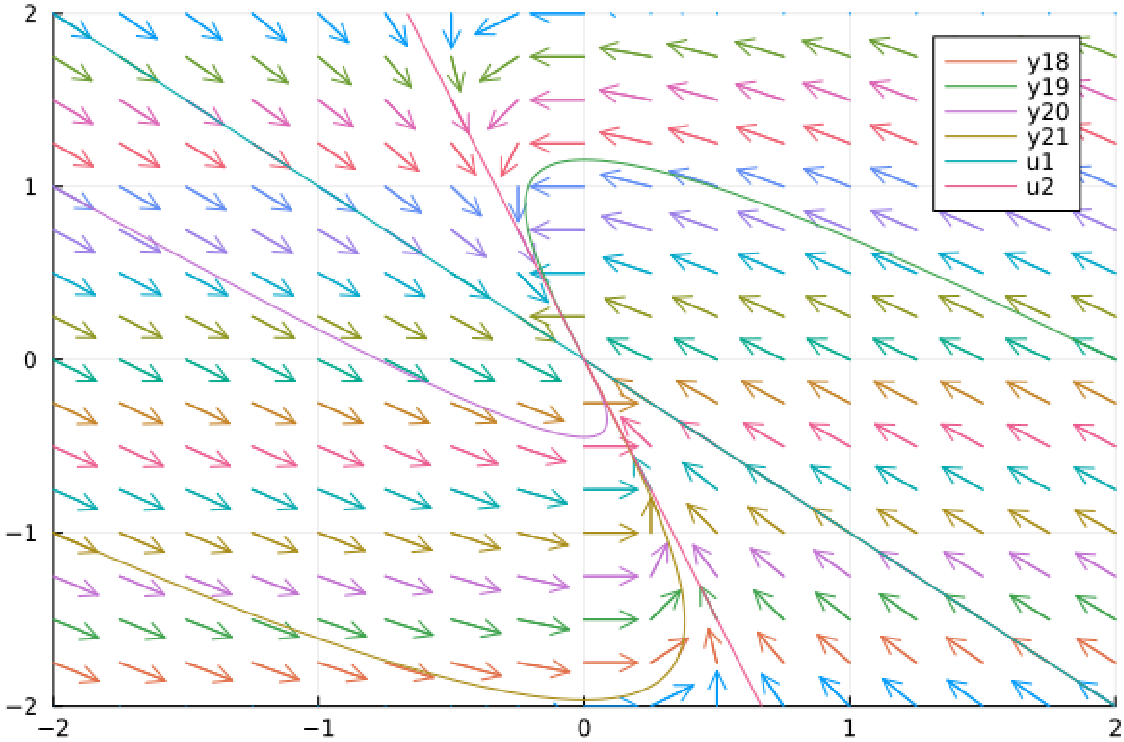
In [6]:
```
U=eigvecs(A)
```

Out[6]:
```
2×2 Matrix{Float64}:
 -0.707107   0.316228
  0.707107  -0.948683
```

The eigenvalues $\lambda_1 = -3$ and $\lambda_2 = -1$ are negative, so the fixed point at the origin is stable. Now add the eigenvectors u_1 and u_2 to the plot.

In [7]:
```
v1(t)=U[:,1]*t
v2(t)=U[:,2]*t
plot!(t->v1(t)[1],t->v1(t)[2],-5:0.1:5,
 xlim=(-2,2),ylim=(-2,2),label="u1")
plot!(t->v2(t)[1],t->v2(t)[2],-5:0.1:5,
 xlim=(-2,2),ylim=(-2,2),label="u2")
```

Out[7]:



In [ ]: