Example 5.4. Consider the nonlinear RLC circuit with $L = 1$, $C = 1$, and v-i characteristic $f(x) = x^3 - x$. Determine the behavior of this circuit over time.

The linearized equation is $dX/dt = AX$ where

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}.$$

In [1]:
```
A=[1 -1; 1 0]
```

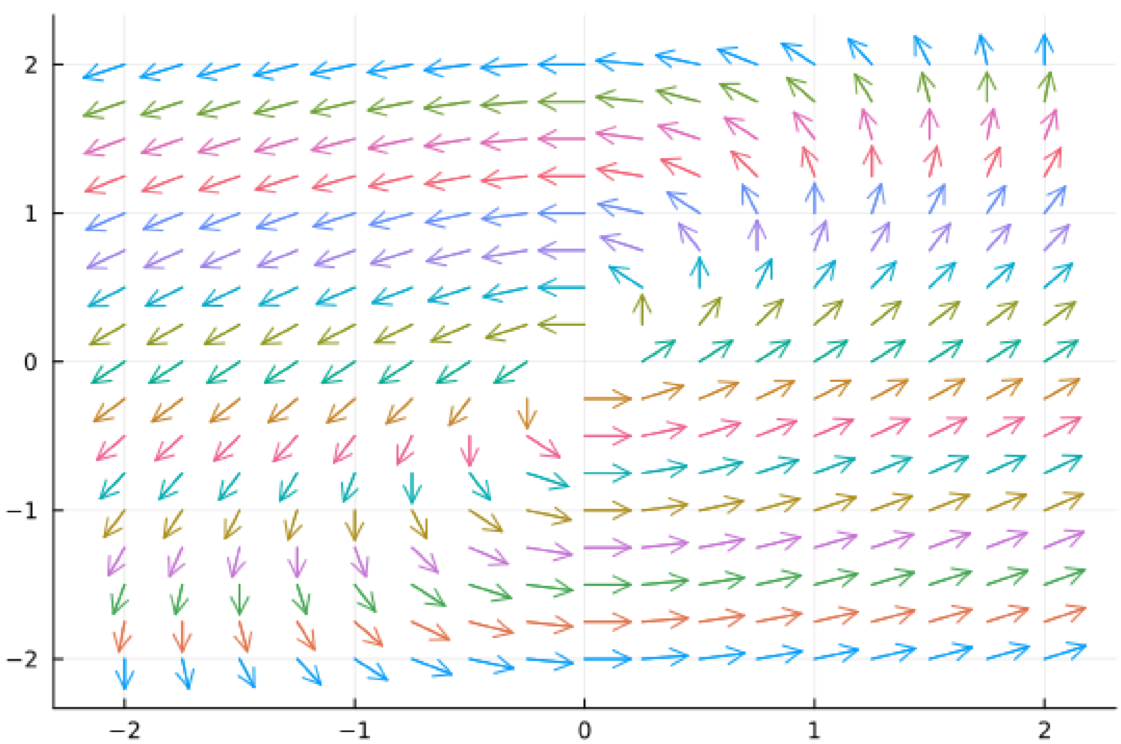
Out[1]:
```
2×2 Matrix{Int64}:
 1  -1
 1   0
```

In [2]:
```
using Plots, LinearAlgebra
```

The direction field for the linearized equation.

In [3]:
```
xs=-2:0.25:2
ys=-2:0.25:2
quiver(
xs*ones(length(ys))',
ones(length(xs))*ys',
quiver=(x,y)->A*[x,y]/norm(A*[x,y])*0.2,
)
```

Out[3]:



In [4]:
```
eigvals(A)
```

Out[4]:
```
2-element Vector{ComplexF64}:
 0.5 - 0.8660254037844385im
 0.5 + 0.8660254037844385im
```

In [5]:
```
U=eigvecs(A)
```

Out[5]:
```
2×2 Matrix{ComplexF64}:
 0.353553-0.612372im  0.353553+0.612372im
 0.707107-0.0im       0.707107+0.0im
```

Solutions to the linearized equation are

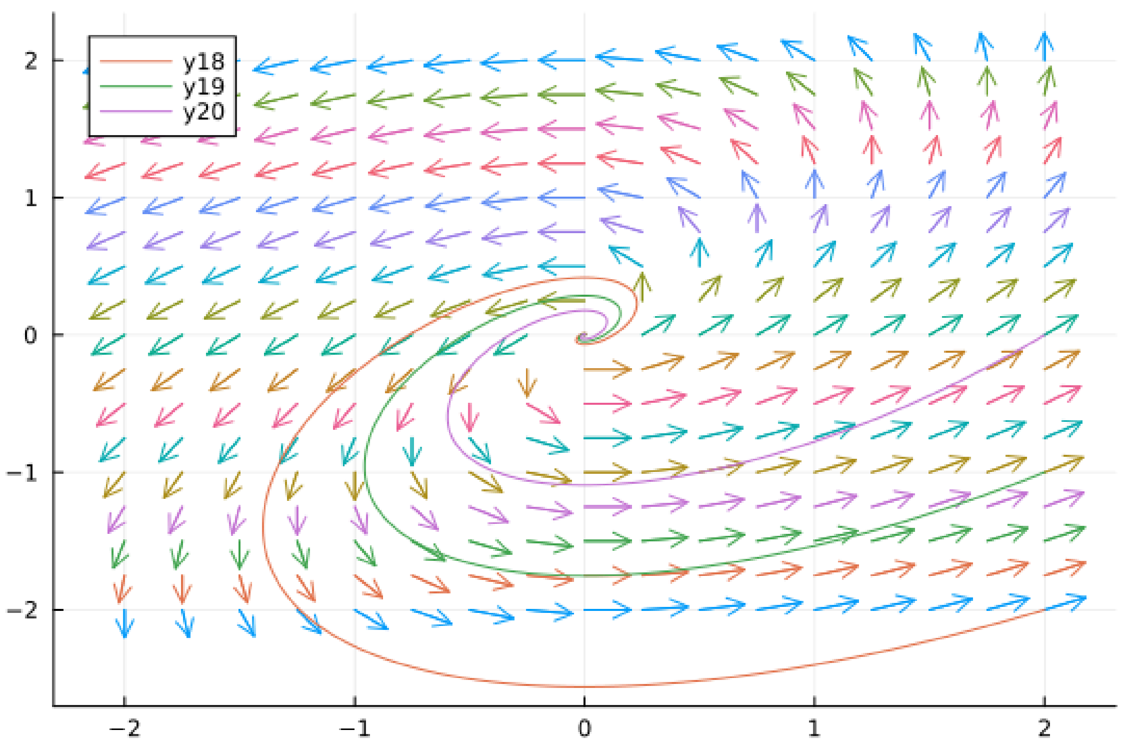
$$X(t) = e^{At}c$$

where c is the initial condition at time $t = 0$.

Plot some representative solutions. Since the real part of the eigenvalues is $1/2$ and greater than 0 the fixed point at the origin is unstable. Plot the solution backwards in time.

In [6]:
```
X(t)=exp(A*t)*[2,-2]
plot!(t->X(t)[1],t->X(t)[2],0:-0.01:-50)
X(t)=exp(A*t)*[2,-1]
plot!(t->X(t)[1],t->X(t)[2],0:-0.01:-50)
X(t)=exp(A*t)*[2,0]
plot!(t->X(t)[1],t->X(t)[2],0:-0.01:-50)
```

Out[6]:



Nonlinear direction field

In [7]:
```
L=1
C=1
F(i,vc)=[1/L*(-vc-i^3+i),i/C]
```

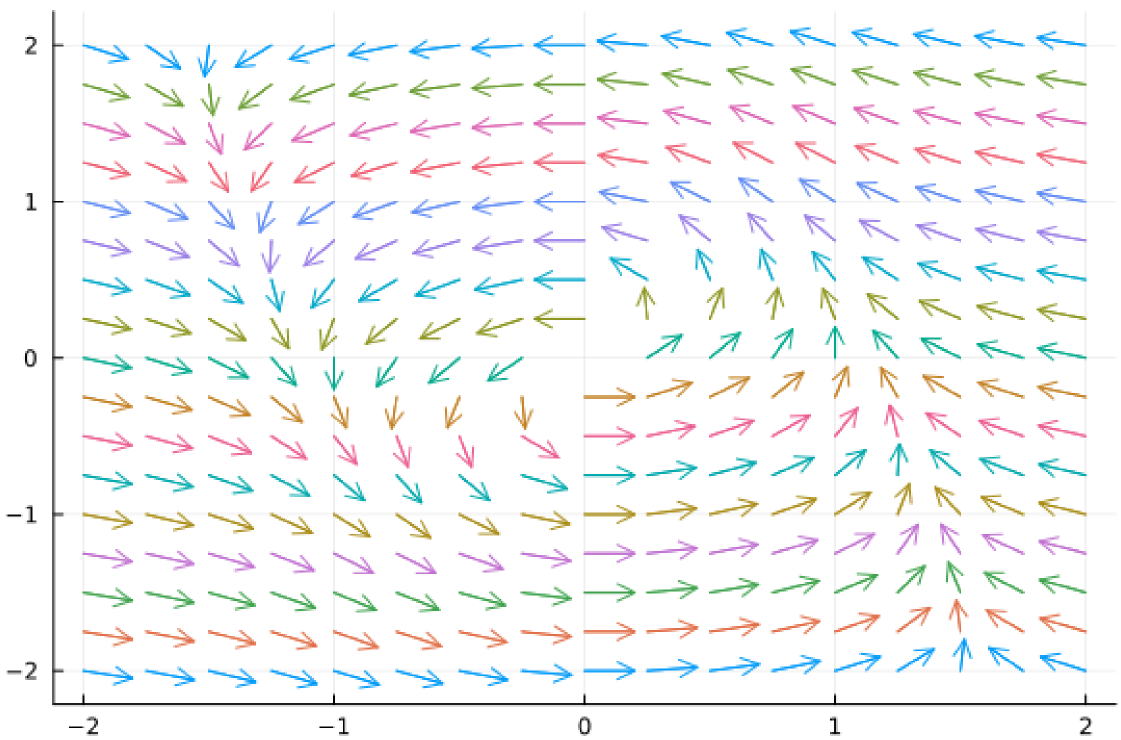
Out[7]:
```
F (generic function with 1 method)
```

In [8]:
```
F(0,0)
```

Out[8]:
```
2-element Vector{Float64}:
 0.0
 0.0
```

In [9]:
```
xs=-2:0.25:2
ys=-2:0.25:2
quiver(
xs*ones(length(ys))',
ones(length(xs))*ys',
quiver=(x,y)->F(x,y)/norm(F(x,y))*0.2,
)
```

Out[9]:



Note the direction of the arrows for the nonlinear direction field point in the opposite direction as the linearized system. This can happen because the linearized system only represents the non-linear system near the fixed point at the origin. In particular, the arrows near the origin for direction fields of the the linearized and nonlinear systems are similar.

In [ ]: