Starting in Step 3

Absolute error is related to digits after the decimal point

$$e_{abs} = |x - x^*|$$

decimal places has

$$e_{abs} \leq 0.5 \times 10^{-n}$$

abbreviate as 3D for 3 digits after the decimal point

$n = 3$

Relative error is related to number of significant digits

$$e_{rel} \approx \frac{e_{abs}}{|x^*|}$$

o n significant digits has

$$e_{rel} \leq 5 \times 10^{-n}$$

abbreviate 3S for three significant digits

$n = 3$

propagation of error:

$x \in \mathbb{R}$ approximation to x called x^*

$y \in \mathbb{R}$ y called y^*

$e_{abs}(x) = |x - x^*|$ $\quad e_{abs}(y) = |y - y^*|$

What is the error in $x+y$?

by definition exact answer: $z = x+y$

assume I can add x^* and y^* exactly then propagated error is

$$e_{abs}(x+y) = |x+y-(x^*+y^*)| \leq |x-x^*| + |y-y^*| \leq e_{abs}(x) + e_{abs}(y)$$

Suppose we're working with 3D rounding

3 digits after the decimal point.

$x = 1.2345678$ $\quad x^* = 1.235$

$y = \pi$ $\quad y^* = 3.142$

```
julia> 1.2345678-1.235
-0.0004322000000001047

julia>
```

$$e_{abs}(x) = |1.2345678 - 1.235| = |-0.0004322| = .0004322 \leq 0.0005$$

okay...

$e_{abs} \leq 0.5 \times 10^{-n}$ ← $n=3$

Check the estimate on digits ...

$0.5 \times 10^{-3} = 0.0005$

```
julia> π-3.142
-0.0004073464102067881

julia> # type \pi<tab> to get the π
```

$e_{abs}(y) = |y - y^*| = |\pi - 3.142|$

$= |-.000407\ldots| = .000407 \leq .0005$

↑ bound for 3-digits after the decimal...

$x+y = 1.2345678 + \pi$

$x^*+y^* = 1.235 + 3.142 = 4.377$ ← no accurate to the third decimal place anymore

didn't do any rounding after adding so didn't generate any new error...

$e_{abs}(x+y) = |1.2345678+\pi - 4.377|$

```
julia> 1.2345678+π-4.377
-0.0008395464102068928

julia>
```

$= |-.0008395\ldots| \not\leq 0.0005$

so not accurate to three decimal places...

$\leq e_{abs}(x) + e_{abs}(y)$

↑ actually equal to the sum... in this case because the errors didn't cancel.

idea when reporting the answer only write 3.38 which we know is correctly rounded...

By default the computer will print out lots of digits, even if the error estimates indicate that they are not likely to be correct...

Until next time read Example in step 3 on page 12...which goes through two more cases, the second for relative error.