$ax^2 + bx + c = 0$

where b is large...

$a = 1, \quad b = 10{,}000 \quad c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10000 \pm \sqrt{10000^2 - 8}}{2}$$

```
julia> sqrt(10000^2-8)
9999.999599999992
```

$$\approx \frac{-10000 + \overbrace{9999.999600}^{\text{10 sig digits}}}{2}$$

note, if I rounded more numbers would cancel

The plus solution

$$\frac{-.0004}{2} = -.0002$$

only one significant digit

alternative formula

$$\frac{-2c}{b + \sqrt{b^2 - 4ac}} = \frac{-4}{10000 + \underbrace{9999.999600}_{\text{10 sig digits}}}$$

$$= \frac{-4}{19999.9996} \approx -.\underbrace{00020000000040}_{\text{10 sig digits...}}$$

```
julia> -4/19999.9996
-0.00020000000400000007
```

$a = 1, \quad b = 10{,}000 \quad c = 2$

What about 5 sig digit?

$$x = \frac{-10000 \pm \sqrt{10000^2 - 8}}{2} \simeq \frac{-10000 + 10000}{2} \simeq 0$$

$$x = \frac{-4}{10000 + \sqrt{10000^2 - 8}} = \frac{-4}{20000} = -.0002$$

Note, it's not that solving quadratic equations is so exciting, but that this kind of cancellation that spoils the answer is something to look out for in other problems...

Another example... approximating derivatives...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Idea... take h small and approximate

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \quad \text{for } h = 0.00001$$

That's the subtraction of two nearly equal numbers...

Example

How about

$$f(x) = \sqrt{x}$$

$$f'(x) \approx \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$f'(1) \approx \frac{\sqrt{1.00001} - \sqrt{1}}{0.00001}$$

loss of precision in the numerator from subtracting two nearly equal numbers...

One can find the derivate in this case exactly so there is no need to approximate. But what if one simply wanted to accurately computer the quantity

$$\frac{\sqrt{1.00001}-\sqrt{1}}{0.00001}$$

directly plug it in...

```
julia> (sqrt(1.00001)-1)/0.00001
0.4999987500031721
```

These digits

can't both be correct...

alternative... simplify...

$$\left(\frac{\sqrt{1.00001}-\sqrt{1}}{0.00001}\right)\left(\frac{\sqrt{1.00001}+\sqrt{1}}{\sqrt{1.00001}+\sqrt{1}}\right)$$

$$= \frac{1.00001-1}{(0.00001)(\sqrt{1.00001}+\sqrt{1})} = \frac{0.00001}{(0.00001)(\sqrt{1.00001}+\sqrt{1})}$$

Suspect this is more accurate... $\rightarrow$

```
julia> 1/(sqrt(1.00001)+1)
0.49999875000625
```

These are different...

This was better

- One idea... plug the answer in and see how well it satisfies the original problem...
- You obtain a residual and then need to infer the error is...

Another Idea... In this case, we can simply work with more digits of precision and see if the answer changes...

```
julia> 1/(sqrt(big(1.00001))+1)
0.49999875000624995274885029509666478455015961
53260290142123179309182281226896313
```

oops

seems more difficult

```
julia> (sqrt(big(1.00001))-1)/big(0.00001)
0.49999875000952550567031556805649459125914081
18754744962212802858517625602573717
```

It seems that 1.00001 does not accurately reflect 1+h so instead seperate out the h as a seperate quantity

still loss of precision, but more digits to start with.

h h

```
julia> (sqrt(1+big(0.00001))-1)/big(0.00001)
0.49999875000624996093767118265437091173288046
49729270639816635610802638820053441
```

different, again less accurate

These two now match to the digits from the calculation we suspected to be more accurate...

also h

addition, so no loss of precision...

```
julia> 1/(sqrt(1+big(0.00001))+1)
0.49999875000624996093767118265437091173288046
49729270639816635610802638816400212
```

different, more accurate

This calculation

```
julia> 1/(sqrt(1.00001)+1)
0.49999875000625
```

was actually accurate to all the displayed digits. In part because of luck, but mostly because there was no cancelation that caused loss of precision...