

Newton's method: Let $e_n = x_n - p$.

Thus $e_{n+1} = \frac{e_n^2}{2} \frac{f''(\xi_n)}{f'(x_n)}$ This is the quadratic convergence

We already know $x_n \rightarrow p$ as $n \rightarrow \infty$

This means $e_n \rightarrow 0$ as $n \rightarrow \infty$

Also ξ_n is between p and x_n for each n .

Since $x_n \rightarrow p$ and ξ_n is trapped between them then $\xi_n \rightarrow p$ as $n \rightarrow \infty$.

after iterating for some time $f''(\xi_n) \approx f''(p)$

and $f'(x_n) \approx f'(p) \neq 0$.

Thus $e_{n+1} \approx \frac{f''(p)}{2f'(p)} e_n^2 = M e_n^2$ where $M = \frac{f''(p)}{2f'(p)}$

That is $e_{n+1} \approx M e_n^2$ quadratic convergence...