Vandermonde Matrices and fitting a polynomial to data using least squares.

basis for polynomials...

$$p(x) = c_1 \varphi_1(x) + c_2 \varphi_2(x) + \cdots + c_n \varphi_n(x)$$

but could be any linearly independent collection of function.

Given the data (x_i, y_i) for $i = 1, \dots, m$, I want to solve for the c's such that

$$J(c) = \sum_{i=1}^{m} \| y_i - p(x_i) \|^2$$

is minimized.

The Vandermonde Matrix

$$V = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_n(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_n(x_2) \\ \vdots & \vdots & & \vdots \\ \varphi_1(x_m) & \varphi_2(x_m) & & \varphi_n(x_m) \end{bmatrix}$$

Then

$$Vc = \begin{bmatrix} p(x_1) \\ p(x_2) \\ \vdots \\ p(x_m) \end{bmatrix} \approx \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$$

Solve least squares problem $Vc = y$ minimize $\|Vc - y\|$.

Julia... $c = V \backslash y$