

- Review all checkpoint questions from Steps 1-13, 16-23, 26-27, 29-33.
- Review homeworks 1 to 5 and be prepared to perform simple calculations related to
  - Significant digits, relative error, rounding error.
  - Propagated error, generated error.
  - Bisection, secant and Newton methods.
  - Matrix norms  $\|A\|_\infty$  and  $\|A\|_1$ .
  - How to create a difference table.
  - Use a difference table to find an interpolating polynomial.
- 1 • Be able to state Newton's method for solving  $f(x)=0$ .
- 2 • Know the statement and proof of Taylor's Theorem.
- 7 3 • ~~Know the statement and proof of the polynomial interpolation theorem.~~
- 4 • Know the proof of the rate of convergence of Newton's method.
- 5 • State the Gauss-Seidel method for solving  $Ax=b$  and explain what kinds of matrices  $A$  are appropriate for using this method.
- 6 • State the power method for finding the eigenvalue of largest magnitude and a corresponding eigenvector.
- 7 • The definition of the Lagrange polynomial basis functions.  $l_j(x) = \frac{\prod_{i \neq j} (x-x_i)}{\prod_{i \neq j} (x_j-x_i)}$
- 8 • The definition of the condition number of a matrix.
- 1 9 • Know how to use the condition number for backward error analysis.
- 4 10 • Know how to prove that the matrix norm  $\|A\|_2 = \max\{ \|Ax\|_2 : \|x\|_2 \leq 1 \}$  corresponding to the Euclidian norm is given by  $\|A\|_2 = \max\{ \lambda^{1/2} : \lambda \text{ is an eigenvalue of } A^T A \}$ .
- 8 11 • ~~Given a matrix  $A$  find the Householder reflector for the first step in the QR factorization.~~  $H = I - 2v v^T$   $v = \frac{a_1 - c e_1}{\|a_1 - c e_1\|}$   $c = \pm \|a_1\|$
- 12 • State Simpson's method for approximating a definite integral.
- 3 13 • Prove the  $n$ -point Gauss quadrature method is exact for polynomials of degree  $2n-1$ .
- 14 • Use Taylor series to derive the  $n$ -th order Taylor method.

$$y(x_{n+1}) \approx y(x_n) + h y'(x_n) + \frac{h^2}{2} y''(x_n) + \frac{h^3}{6} y'''(x_n)$$

$$y' = f(x, y)$$

$$y'' = \frac{d}{dx} f(x, y) = f_x(x, y) + f_y(x, y) f(x, y)$$

$$y''' = \frac{d}{dx} (f_x(x, y) + f_y(x, y) f(x, y))$$