Math 466/666: Homework 5 Version A

This homework is based on steps 28 through 30 from the text *First Steps in Numerical Analysis* by Hosking, Joe, Joyce and Turner. Students are encouraged to work in groups and consult resources outside of the required textbook when doing the homework for this class. Please cite any additional sources you used to complete your work.

1. Use the trapezoidal method with $h = 1, 0.5$ and 0.25 to estimate

$$\int_0^1 \frac{1}{1+x^2}dx.$$

2. Estimate

$$\int_0^1 \frac{1}{1+x^2}dx$$

to 4 digits after the decimal point (4D) using Simpson's rule.

3. Consider using the Gauss quadrature to evaluate

$$\int_0^1 \frac{1}{1+u^2}du.$$

(i) First make the change of variables $x = 2u - 1$ to transform the integral as

$$\int_0^1 \frac{1}{1+u^2}du = \int_{-1}^1 f(x)dx.$$

What is $f(x)$?

(ii) Apply the two-point Gauss quadrature formula

$$\int_{-1}^1 f(x)dx \approx f(-1/\sqrt{3}) + f(1/\sqrt{3})$$

to approximate the integral.

(iii) Apply the three-point Gauss quadrature formula

$$\int_{-1}^1 f(x)dx \approx \frac{5}{9}f(-\sqrt{3/5}) + \frac{8}{9}f(0) + \frac{5}{9}f(\sqrt{3/5})$$

to approximate the integral.

(iv) [Extra Credit and Math 666] Apply the four-point Gauss quadrature formula to approximate

$$\int_0^1 \frac{1}{1+u^2}du.$$

Comment on the accuracy compared to the trapezoid method and Simpson's rule.

4. [Extra Credit and Math 666] Use the integral remainder form of Taylor's theorem

$$f(x+\delta) = f(x) + \delta f'(x) + \frac{\delta^2}{2}f''(x) + \frac{\delta^3}{3!}f'''(x) + \int_x^{x+\delta} \frac{(x+\delta-t)^3}{3!}f^{(4)}(t)dt$$

to rigorously bound the error in Simpson's method.