

Verify Quad order on convergence for Newton

$$|e_{n+1}| \approx \mu |e_n|^2$$

Recall  $e_n = x_n - \xi$  but what's  $\xi$ .

• One idea... run Newton to convergence to find  $\xi$ .  
and then go back to check the order.

•  $e_n \approx x_n - x_{n+1}$  in comparison to  $x_n$  the  
next iterata is close enough  
to see the order...

• Try to verify the  
 $|e_{n+1}| \approx \mu |e_n|^2$

$$\log |e_{n+1}| \approx \log \mu + 2 \log |e_n|$$

When  $e_n$  is  
small  $\log |e_n| \rightarrow -\infty$

$$2 \approx \frac{\log |e_{n+1}| - \log \mu}{\log |e_n|} \leftarrow \text{negligible as } n \rightarrow \infty$$

$$2 \approx \frac{\log |e_{n+1}|}{\log |e_n|} \approx \frac{\log |x_{n+1} - x_{n+2}|}{\log |x_n - x_{n+1}|} \text{ for large } n$$

- Compute this and check it's close to zero.
- Use arb precision so can check a number of iterations.