$$e^z = 1 + z + \frac{1}{2!}z^2 + \frac{1}{3!}z^3 + \frac{1}{4!}z^4 + \dots$$

$$\frac{e^z - 1}{z} = \frac{\left(1 + z + \frac{1}{2!}z^2 + \frac{1}{3!}z^3 + \frac{1}{4!}z^4 + \dots\right) - 1}{z}$$

$$= \frac{z + \frac{1}{2!}z^2 + \frac{1}{3!}z^3 + \frac{1}{4!}z^4 + \dots}{z} = \underbrace{1 + \frac{1}{2!}z + \frac{1}{3!}z^2 + \frac{1}{4!}z^3 + \dots}_{\text{nothing being subtracted any more... no loss of precision ...}}$$

```
julia> f(z)=1+1/2*z+1/6*z^2+1/24*z^3
f (generic function with 1 method)

julia> g(z)=(exp(z)-1)/z
g (generic function with 1 method)
```

```
julia> h(z)=expm1(z)/z
h (generic function with 1 method)
```

(built in)

```
julia> g(0.1)
1.0517091807564771
```

```
julia> f(0.1)
1.0517083333333335
```

almost

```
julia> h(0.1)
1.0517091807564762
```

```
julia> g(0.0001)
1.0000500016667141
```

```
julia> f(0.0001)
1.0000500016667084
```

```
julia> h(0.0001)
1.0000500016667084
```

```
julia> g(0.0000001)
1.0000000494336803
```

```
julia> f(0.0000001)
1.0000000500000017
```

```
julia> h(0.0000001)
1.0000000500000017
```

```
julia> g(0.0000000001)
1.000000082740371
```

```
julia> f(0.0000000001)
1.00000000005
```

```
julia> h(0.0000000001)
1.00000000005
```

$$\cos z = 1 - \frac{z^2}{2} + \frac{z^4}{4!} - \frac{z^6}{6!} + \frac{z^8}{8!} - \cdots$$

$$\frac{\cos z - 1}{z^2} = \frac{\left(1 - \frac{z^2}{2} + \frac{z^4}{4!} - \frac{z^6}{6!} + \frac{z^8}{8!} - \cdots\right) - 1}{z^2}$$

$$= -\frac{1}{2} + \frac{z^2}{4!} - \frac{z^4}{6!} + \frac{z^6}{8!} - \cdots$$

$\cos^2 a + \sin^2 a = 1$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

set $a=b$ we get half angle formulas.

$$\cos(2a) = \cos^2 a - \sin^2 a = \cos^2 a + \sin^2 a - 2\sin^2 a$$

$$= 1 - 2\sin^2 a$$

set $a = z/2$ so $\cos z = 1 - 2\sin^2(z/2)$

$$\frac{\cos z - 1}{z^2} = \frac{\cancel{1} - 2\sin^2(z/2) - \cancel{1}}{z^2} = \frac{-2\sin^2(z/2)}{z^2}$$

```
julia> f(z)=-1/2+z^2/24-z^4/720+z^6/40320
f (generic function with 1 method)

julia> g(z)=(cos(z)-1)/z^2
g (generic function with 1 method)

julia> f(0.1)
-0.4995834721974206

julia> g(0.1)
-0.49958347219741783

julia> h(z)=-2*sin(z/2)^2/z^2
h (generic function with 1 method)

julia> h(0.1)
-0.49958347219742333
```

```
help?>
```

type ? to get to help

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$-\frac{2\sin^2(z/2)}{z^2} = -2\left[\frac{\sin(z/2)}{z}\right]^2 = -2\left(\frac{\sin\left(\pi\left(\frac{z}{2\pi}\right)\right)}{2\cdot\pi\, z/2\pi}\right)^2$$

$$= -2\left(\frac{\mathrm{sinc}(z/2\pi)}{2}\right)^2 = -\frac{1}{2}\,\mathrm{sinc}^2(z/2\pi)$$

```
julia> q(z)=-1/2*sinc(z/(2*pi))^2
q (generic function with 1 method)

julia> q(0.1)
-0.49958347219742355
```

Linear Algebra...

① Solving $Ax=b$ (2.1) - (2.16

② Eigenvalue - eigenvector problem: (2.17) - (2.20)

From Math 330:

Solve $Ax=b$

- By Gaussian elimination: Row operations on the augmented matrix $[A \,|\, b]$
  - elimination step $r_i \leftarrow r_i - \alpha r_j \quad i \neq j$
  - rescaling $r_i \leftarrow \beta r_i \quad \beta \neq 0$
  - row swap $r_i \leftrightarrow r_j \quad i \neq j$
- LU factorization. Only use elimination steps to create a row echelon form of A. Call that U. Then put the α's from each $r_i \leftarrow r_i - \alpha r_j$ in slot (i,j) of the L matrix, Put 1's on the diagonal of L.

- PLU factorization: Uses row swap as well and P is a permutation matrix to undo the row swaps to get back to A.

- QR factorization: Use Gram-schmidt orthogonalization

$$c_i \leftarrow c_i - \alpha c_j \qquad j \neq i$$
$$c_i \leftarrow \beta c_i$$

Put the orthonormal vectors as columns in Q and R contains $1/\beta$ on the diagonal and α at (j,i) above the diagonal.

- Singular value decomposition $A = U \Sigma V^T$. Was related to the eigenvalues and eigenvectors of $B = A^T A$.

Solve for eigenvalues

- Theory of determinants to solve $Au = \lambda u$

  u is eigenvector λ is eigenvalue.