```
> restart;
> eq:=D(y)=(s->f(s,y(s)));
```
$$eq := \mathrm{D}(y) = (s \mapsto f(s, y(s))) \quad (1)$$
```
> yp:=y(xn-h)+2*h*f(xn,y(xn));
```
$$yp := y(xn - h) + 2\,h\,f(xn, y(xn)) \quad (2)$$
```
> ynp1:=y(xn)+h/12*(-f(xn-h,y(xn-h))+8*f(xn,y(xn))+5*f(xn+h,yp));
```
$$ynp1 := y(xn) + \frac{1}{12}\left(h\left(-f(xn-h, y(xn-h)) + 8\,f(xn, y(xn)) + 5\,f(xn+h, y(xn - h) + 2\,h\,f(xn, y(xn)))\right)\right) \quad (3)$$
```
> r:=y(xn+h)-ynp1;
```
$$r := y(xn+h) - y(xn) - \frac{1}{12}\left(h\left(-f(xn-h, y(xn-h)) + 8\,f(xn, y(xn)) + 5\,f(xn + h, y(xn-h) + 2\,h\,f(xn, y(xn)))\right)\right) \quad (4)$$
```
> subs(h=0,r);
```
$$0 \quad (5)$$
```
> T1:=diff(r,h);
```
$$T1 := \mathrm{D}(y)(xn+h) + \frac{f(xn-h, y(xn-h))}{12} - \frac{2\,f(xn, y(xn))}{3} - \frac{5\,f(xn+h, y(xn-h) + 2\,h\,f(xn, y(xn)))}{12} - \frac{1}{12}\Big(h\big(\mathrm{D}_1(f)(xn-h, y(xn - h)) + \mathrm{D}_2(f)(xn-h, y(xn-h))\,\mathrm{D}(y)(xn-h) + 5\,\mathrm{D}_1(f)(xn+h, y(xn-h) + 2\,h\,f(xn, y(xn))) + 5\,\mathrm{D}_2(f)(xn+h, y(xn-h) + 2\,h\,f(xn, y(xn)))\,(-\mathrm{D}(y)(xn-h) + 2\,f(xn, y(xn)))\big)\Big) \quad (6)$$
```
> dr:=eval(subs(eq,T1));
```
$$dr := f(xn+h, y(xn+h)) + \frac{f(xn-h, y(xn-h))}{12} - \frac{2\,f(xn, y(xn))}{3} - \frac{5\,f(xn+h, y(xn-h) + 2\,h\,f(xn, y(xn)))}{12} - \frac{1}{12}\Big(h\big(\mathrm{D}_1(f)(xn-h, y(xn - h)) + \mathrm{D}_2(f)(xn-h, y(xn-h))\,f(xn-h, y(xn-h)) + 5\,\mathrm{D}_1(f)(xn+h, y(xn - h) + 2\,h\,f(xn, y(xn))) + 5\,\mathrm{D}_2(f)(xn+h, y(xn-h) + 2\,h\,f(xn, y(xn)))\,(-f(xn-h, y(xn-h)) + 2\,f(xn, y(xn)))\big)\Big) \quad (7)$$
```
> subs(h=0,dr);
```
$$0 \quad (8)$$
```
> ddr:=eval(subs(eq,diff(dr,h)));
```
$$ddr := \mathrm{D}_1(f)(xn+h, y(xn+h)) + \mathrm{D}_2(f)(xn+h, y(xn+h))\,f(xn+h, y(xn+h)) - \frac{\mathrm{D}_1(f)(xn-h, y(xn-h))}{6} - \frac{\mathrm{D}_2(f)(xn-h, y(xn-h))\,f(xn-h, y(xn-h))}{6} - \frac{5\,\mathrm{D}_1(f)(xn+h, y(xn-h) + 2\,h\,f(xn, y(xn)))}{6} - \frac{1}{6}\big(5\,\mathrm{D}_2(f)(xn+h, y(xn-h) + 2\,h\,f(xn, y(xn)))\,(-f(xn-h, y(xn-h)) + 2\,f(xn, y(xn)))\big) - \frac{1}{12}\Big(h\big(-\mathrm{D}_{1,1}(f)(xn-h, y(xn-h)) - \mathrm{D}_{1,2}(f)(xn-h, y(xn-h))\,f(xn - h, y(xn-h)) + \big(-\mathrm{D}_{1,2}(f)(xn-h, y(xn-h)) - \mathrm{D}_{2,2}(f)(xn-h, y(xn \quad (9)$$

$$- h)) f(xn-h, y(xn-h))) f(xn-h, y(xn-h)) + D_2(f)(xn-h, y(xn-h)) ( -D_1(f)(xn-h, y(xn-h)) - D_2(f)(xn-h, y(xn-h)) f(xn-h, y(xn-h))) + 5 D_{1,1}(f)(xn+h, y(xn-h) + 2 h f(xn, y(xn))) + 5 D_{1,2}(f)(xn+h, y(xn - h) + 2 h f(xn, y(xn))) (-f(xn-h, y(xn-h)) + 2 f(xn, y(xn))) + 5 (D_{1,2}(f)(xn+h, y(xn-h) + 2 h f(xn, y(xn))) + D_{2,2}(f)(xn+h, y(xn - h) + 2 h f(xn, y(xn))) (-f(xn-h, y(xn-h)) + 2 f(xn, y(xn)))) (-f(xn - h, y(xn-h)) + 2 f(xn, y(xn))) + 5 D_2(f)(xn+h, y(xn-h) + 2 h f(xn, y(xn))) (D_1(f)(xn-h, y(xn-h)) + D_2(f)(xn-h, y(xn-h)) f(xn-h, y(xn - h)))))$$

```
> subs(h=0,ddr);
```

$$0 \quad (10)$$

```
> d3r:=eval(subs(eq,diff(ddr,h))):
> simplify(subs(h=0,d3r));
```

$$0 \quad (11)$$

```
> d4r:=simplify(eval(subs(eq,diff(d3r,h)))):
> simplify(subs(h=0,d4r));
```

$$-f(xn, y(xn))^3 D_{2,2,2}(f)(xn, y(xn)) + \frac{1}{3}\left(\left(-2 D_{2,2}(f)(xn, y(xn)) D_2(f)(xn, y(xn)) - 9 D_{1,2,2}(f)(xn, y(xn))\right) f(xn, y(xn))^2\right) + \frac{1}{3}\left(\left(7 D_2(f)(xn, y(xn))^3 - 9 D_{2,2}(f)(xn, y(xn)) D_1(f)(xn, y(xn)) + 5 D_{1,2}(f)(xn, y(xn)) D_2(f)(xn, y(xn)) - 9 D_{1,1,2}(f)(xn, y(xn))\right) f(xn, y(xn))\right) + \frac{7 D_1(f)(xn, y(xn)) D_2(f)(xn, y(xn))^2}{3} - 3 D_{1,2}(f)(xn, y(xn)) D_1(f)(xn, y(xn)) + \frac{7 D_2(f)(xn, y(xn)) D_{1,1}(f)(xn, y(xn))}{3} - D_{1,1,1}(f)(xn, y(xn)) \quad (12)$$

```
> f:=(xi,eta)->A*eta;
```

$$f := (\xi, \eta) \mapsto A \eta \quad (13)$$

```
> method:=y(xn+h)=ynp1;
```

$$method := y(xn+h) = y(xn) + \frac{h(-A y(xn-h) + 8 A y(xn) + 5 A (y(xn-h) + 2 h A y(xn)))}{12} \quad (14)$$

```
> ceq:=eval(subs(y=(s->rho^s),method));
```

$$ceq := \rho^{xn+h} = \rho^{xn} + \frac{h\left(-A \rho^{xn-h} + 8 A \rho^{xn} + 5 A \left(\rho^{xn-h} + 2 h A \rho^{xn}\right)\right)}{12} \quad (15)$$

```
> solve(ceq,rho);
```

$$\left(\frac{5 A^2 h^2}{12} + \frac{A h}{3} + \frac{1}{2} + \frac{\sqrt{25 A^4 h^4 + 40 A^3 h^3 + 76 A^2 h^2 + 96 A h + 36}}{12}\right)^{\frac{1}{h}}, \quad (16)$$

$$\left(\frac{5A^2h^2}{12}+\frac{Ah}{3}+\frac{1}{2}-\frac{\sqrt{25A^4h^4+40A^3h^3+76A^2h^2+96Ah+36}}{12}\right)^{\frac{1}{h}}, 0$$

```
> ceq2:=subs({xn=0,h=1},ceq);
```

$$ceq2 := \rho = 1-\frac{A}{12\rho}+\frac{2A}{3}+\frac{5A\left(\frac{1}{\rho}+2A\right)}{12} \quad \textbf{(17)}$$

```
> S:=solve(ceq2,rho);
```

$$S := \frac{5A^2}{12}+\frac{A}{3}+\frac{1}{2}+\frac{\sqrt{25A^4+40A^3+76A^2+96A+36}}{12}, \frac{5A^2}{12}+\frac{A}{3}+\frac{1}{2}-\frac{\sqrt{25A^4+40A^3+76A^2+96A+36}}{12} \quad \textbf{(18)}$$

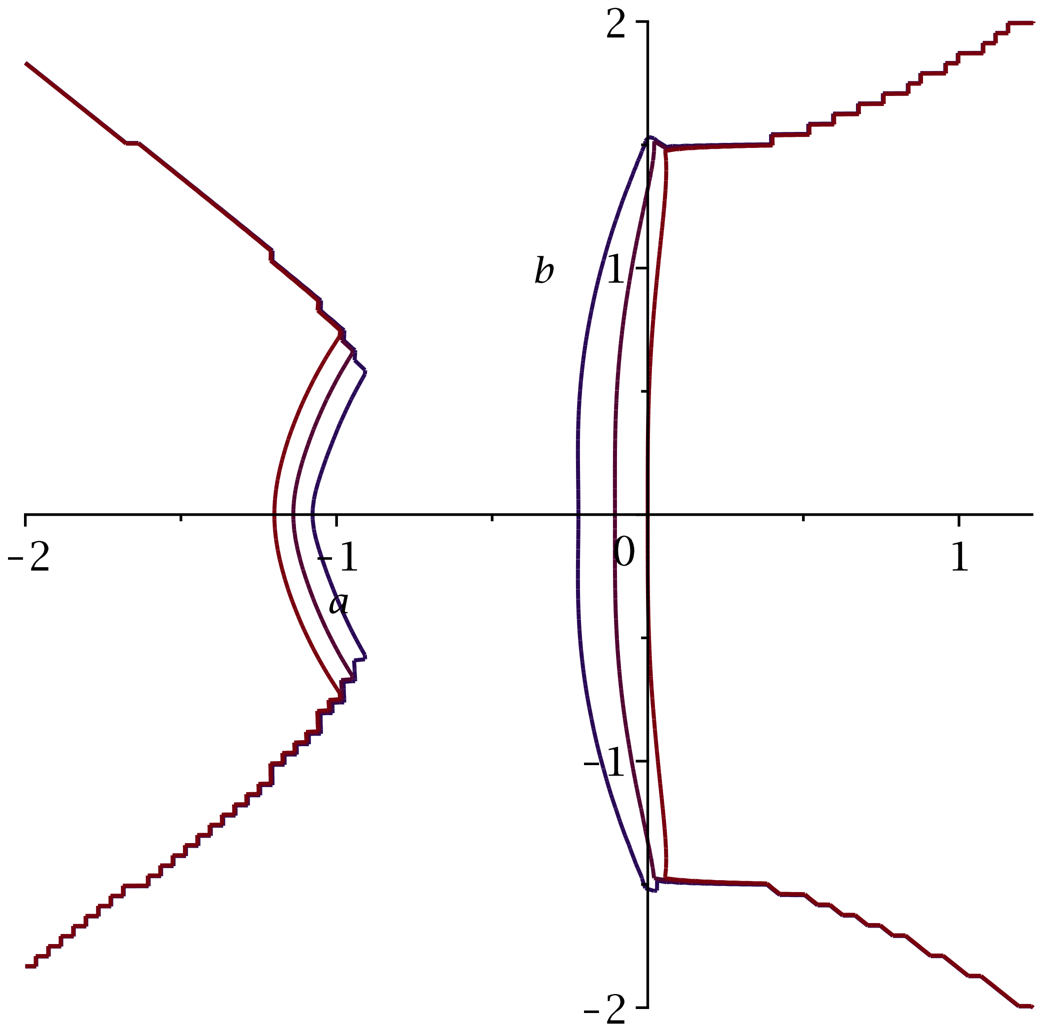
```
> # the linear stability region is all values of A such that
  |rho|<1
> Z1:=subs(A=a+I*b,abs(S[1]));
```

$$Z1 := \left|\frac{5(a+\mathrm{I}b)^2}{12}+\frac{a}{3}+\frac{\mathrm{I}b}{3}+\frac{1}{2}+\frac{\sqrt{25(a+\mathrm{I}b)^4+40(a+\mathrm{I}b)^3+76(a+\mathrm{I}b)^2+96a+96\mathrm{I}b+36}}{12}\right| \quad \textbf{(19)}$$

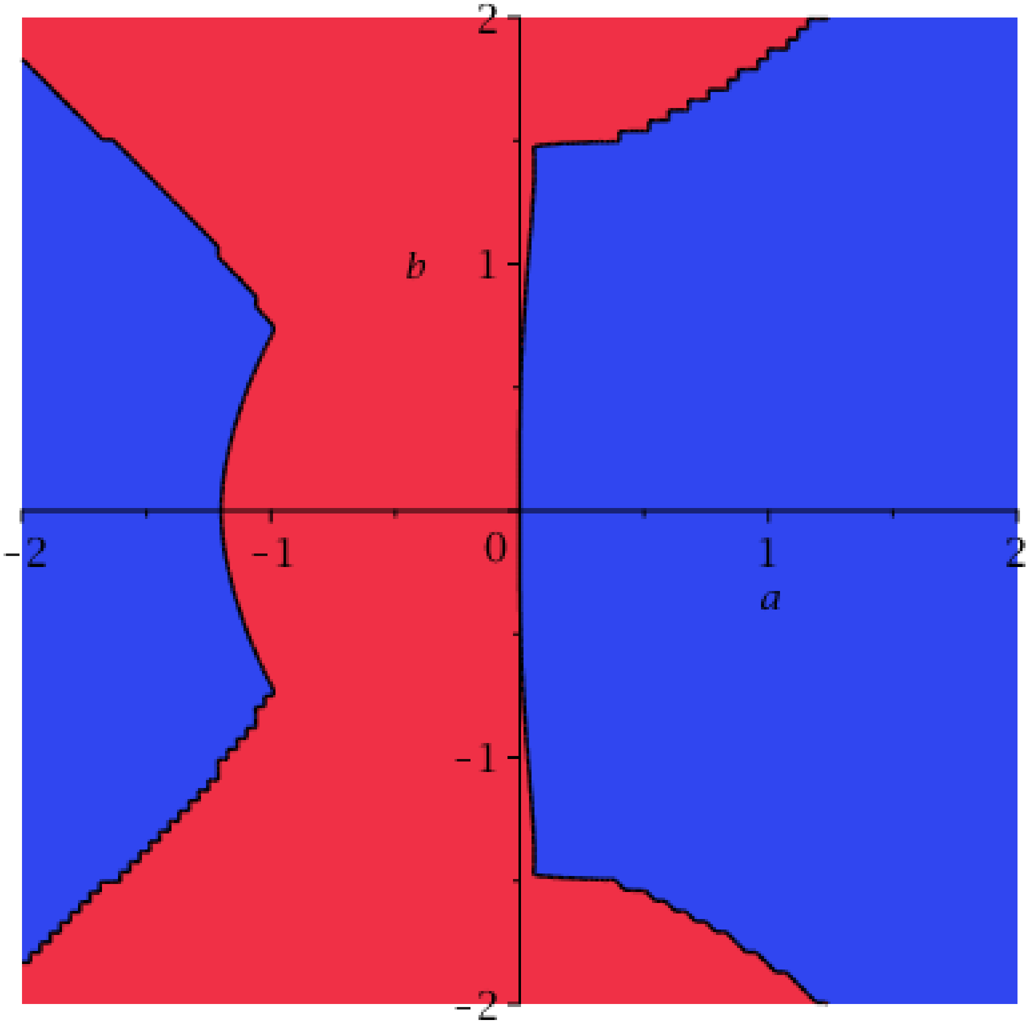
```
> Z2:=subs(A=a+I*b,abs(S[2]));
```

$$Z2 := \left|\frac{5(a+\mathrm{I}b)^2}{12}+\frac{a}{3}+\frac{\mathrm{I}b}{3}+\frac{1}{2}-\frac{\sqrt{25(a+\mathrm{I}b)^4+40(a+\mathrm{I}b)^3+76(a+\mathrm{I}b)^2+96a+96\mathrm{I}b+36}}{12}\right| \quad \textbf{(20)}$$

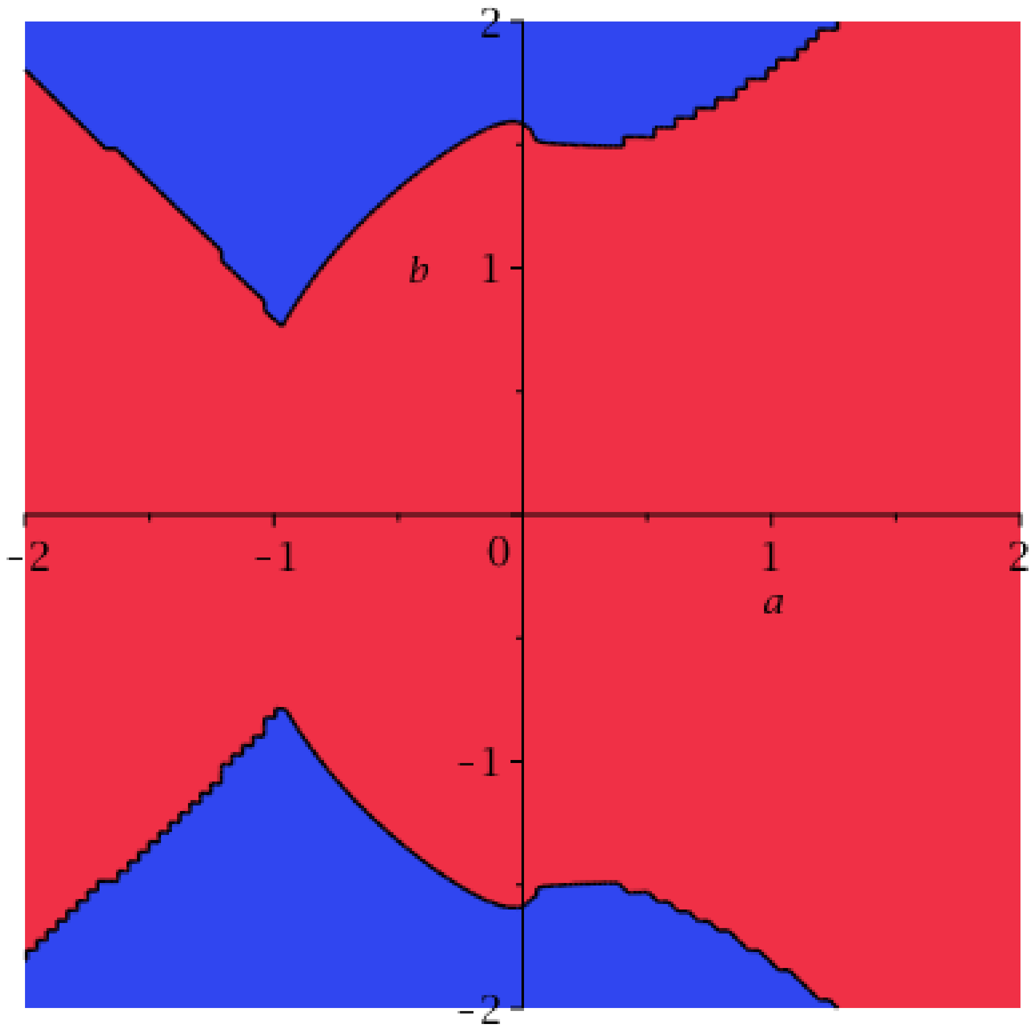
```
> with(plots):
> contourplot(Z1,a=-2..2,b=-2..2,contours=[1,.9,.8],grid=[100,100])
  ;
```



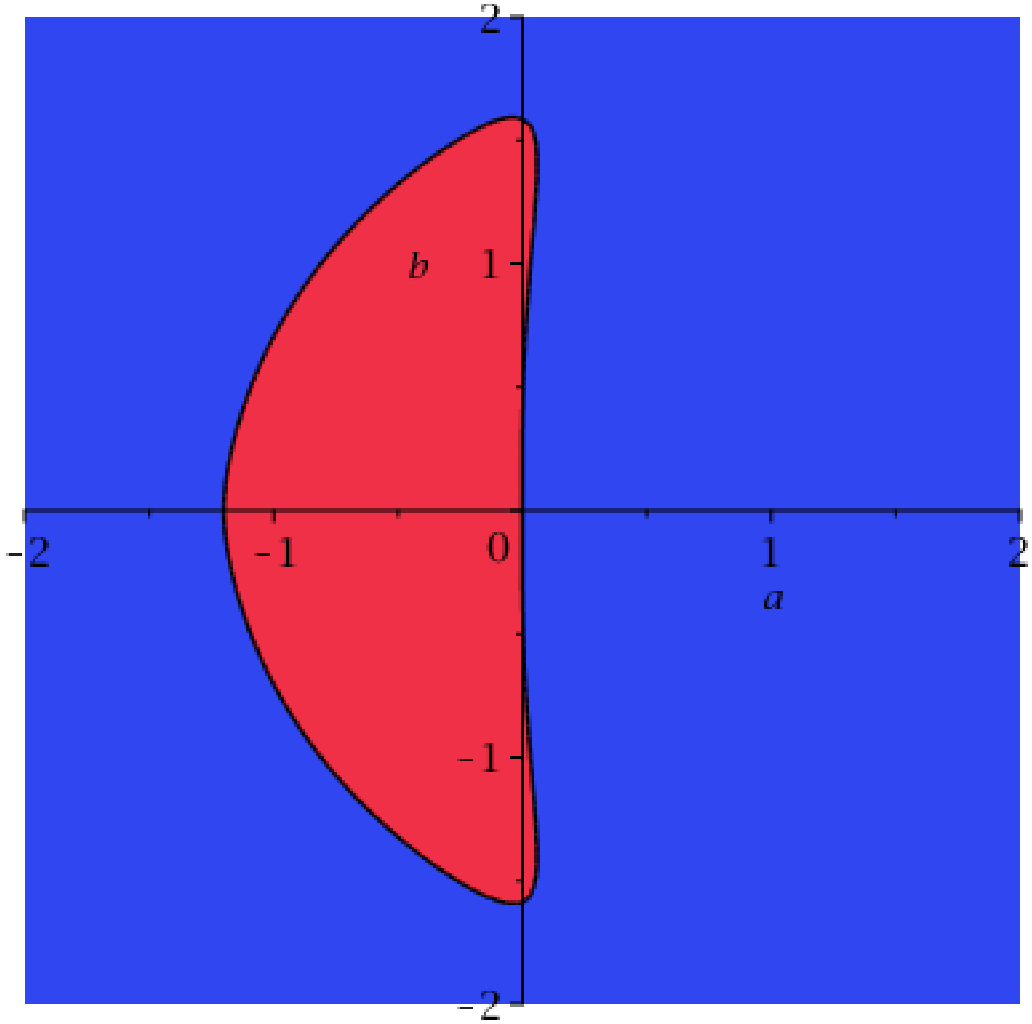

```
> contourplot(Z1,a=-2..2,b=-2..2,contours=[1],grid=[100,100],
  filled=true);
```




```
> contourplot(Z2,a=-2..2,b=-2..2,contours=[1],grid=[100,100],
  filled=true);
```




```
> contourplot(max(Z1,Z2),a=-2..2,b=-2..2,contours=[1],grid=[100,
  100],filled=true);
```



2
1
b
-2
-1
0
1
2
a
-1
-2