In[1]:= `ynp1 = a0 * y[tn] + a1 * y[tn - h] + a2 * y[tn - 2 * h] +`
`    h * (bm1 * y'[tn + h] + b0 * y'[tn] + b1 * y'[tn - h] +`
`        b2 * y'[tn - 2 * h]) +`
`    E5 * h^5 * Derivative[5][y][theta] / 5!`

Out[1]= $a0\, y[tn] + a2\, y[-2\,h + tn] + a1\, y[-h + tn] +$
$h\, (b0\, y'[tn] + b2\, y'[-2\,h + tn] + b1\, y'[-h + tn] + bm1\, y'[h + tn]) + \frac{1}{120}\, E5\, h^5\, y^{(5)}[theta]$

In[2]:= `r = y[tn + h] - ynp1`

Out[2]= $-a0\, y[tn] - a2\, y[-2\,h + tn] - a1\, y[-h + tn] + y[h + tn] -$
$h\, (b0\, y'[tn] + b2\, y'[-2\,h + tn] + b1\, y'[-h + tn] + bm1\, y'[h + tn]) - \frac{1}{120}\, E5\, h^5\, y^{(5)}[theta]$

In[3]:= `eq[0] = r == 0 /. y → Function[x, 1]`
`For[j = 1, j < 6, j++,`
` tmp[j] = r /. y → Function[x, x^j];`
` eq[j] = Coefficient[tmp[j], h^j] == 0;`
` Print[eq[j]]`
`]`

Out[3]= $1 - a0 - a1 - a2 == 0$

$1 + a1 + 2\,a2 - b0 - b1 - b2 - bm1 == 0$

$1 - a1 - 4\,a2 + 2\,b1 + 4\,b2 - 2\,bm1 == 0$

$1 + a1 + 8\,a2 - 3\,b1 - 12\,b2 - 3\,bm1 == 0$

$1 - a1 - 16\,a2 + 4\,b1 + 32\,b2 - 4\,bm1 == 0$

$1 + a1 + 32\,a2 - 5\,b1 - 80\,b2 - 5\,bm1 - E5 == 0$

In[5]:= `S1 = Solve[{eq[0], eq[1], eq[2], eq[3], eq[4], eq[5]}, {a0, bm1, b0, b1, b2, E5}]`

Out[5]= $\left\{\left\{a0 \to 1 - a1 - a2,\ bm1 \to \frac{9 - a1}{24},\ b0 \to \frac{1}{24}(19 + 13\,a1 + 8\,a2),\right.\right.$
$\left.\left. b1 \to \frac{1}{24}(-5 + 13\,a1 + 32\,a2),\ b2 \to \frac{1}{24}(1 - a1 + 8\,a2),\ E5 \to \frac{1}{6}(-19 + 11\,a1 - 8\,a2)\right\}\right\}$

In[6]:= `method = ynp1 /. E5 → 0`

Out[6]= $a0\, y[tn] + a2\, y[-2\,h + tn] + a1\, y[-h + tn] +$
$h\, (b0\, y'[tn] + b2\, y'[-2\,h + tn] + b1\, y'[-h + tn] + bm1\, y'[h + tn])$

In[7]:= `m2 = method /. S1[[1]]`

Out[7]= $(1 - a1 - a2)\, y[tn] + a2\, y[-2\,h + tn] + a1\, y[-h + tn] +$
$h\, \left(\frac{1}{24}(19 + 13\,a1 + 8\,a2)\, y'[tn] + \frac{1}{24}(1 - a1 + 8\,a2)\, y'[-2\,h + tn] +\right.$
$\left.\frac{1}{24}(-5 + 13\,a1 + 32\,a2)\, y'[-h + tn] + \frac{1}{24}(9 - a1)\, y'[h + tn]\right)$

In[8]:= `m3 = m2 /. y' → Function[x, f[x, y[x]]]`

Out[8]= $h \left(\dfrac{1}{24} \left(19 + 13\, a1 + 8\, a2 \right) f[tn, y[tn]] + \dfrac{1}{24} \left(1 - a1 + 8\, a2 \right) f[-2\, h + tn, y[-2\, h + tn]] + \right.$
$\left. \dfrac{1}{24} \left(-5 + 13\, a1 + 32\, a2 \right) f[-h + tn, y[-h + tn]] + \dfrac{1}{24} \left(9 - a1 \right) f[h + tn, y[h + tn]] \right) +$
$\left(1 - a1 - a2 \right) y[tn] + a2\, y[-2\, h + tn] + a1\, y[-h + tn]$

In[9]:= `f = Function[{xi, eta}, A * eta]`

Out[9]= `Function[{xi, eta}, A eta]`

In[10]:= `m4 = y[tn + h] == m3`

Out[10]= $y[h + tn] == \left(1 - a1 - a2 \right) y[tn] + a2\, y[-2\, h + tn] + a1\, y[-h + tn] +$
$h \left(\dfrac{1}{24} A \left(19 + 13\, a1 + 8\, a2 \right) y[tn] + \dfrac{1}{24} A \left(1 - a1 + 8\, a2 \right) y[-2\, h + tn] + \right.$
$\left. \dfrac{1}{24} A \left(-5 + 13\, a1 + 32\, a2 \right) y[-h + tn] + \dfrac{1}{24} A \left(9 - a1 \right) y[h + tn] \right)$

In[11]:= `ceq = m4 /. y → Function[s, rho^s]`

Out[11]= $rho^{h+tn} == \left(1 - a1 - a2 \right) rho^{tn} + a2\, rho^{-2\,h+tn} + a1\, rho^{-h+tn} +$
$h \left(\dfrac{1}{24} A \left(19 + 13\, a1 + 8\, a2 \right) rho^{tn} + \dfrac{1}{24} A \left(1 - a1 + 8\, a2 \right) rho^{-2\,h+tn} + \right.$
$\left. \dfrac{1}{24} A \left(-5 + 13\, a1 + 32\, a2 \right) rho^{-h+tn} + \dfrac{1}{24} A \left(9 - a1 \right) rho^{h+tn} \right)$

In[12]:= `ceq2 = ceq /. {a1 → 1, a2 → 1, tn → 1, h → 1}`

Out[12]= $rho^2 == 1 + \dfrac{5\,A}{3} + \dfrac{1}{rho} + \dfrac{A}{3\,rho} - rho + \dfrac{5\,A\,rho}{3} + \dfrac{A\,rho^2}{3}$

In[13]:= `S2 = Solve[ceq2, rho]`

Out[13]= $\left\{ \{rho \to -1\}, \left\{ rho \to \dfrac{-2\,A - \sqrt{3}\,\sqrt{3 + A^2}}{-3 + A} \right\}, \left\{ rho \to \dfrac{-2\,A + \sqrt{3}\,\sqrt{3 + A^2}}{-3 + A} \right\} \right\}$