In[1]:=
```
(* Question 2 *)
(* first recompute the stuff for Question 1
   to avoid having to type in the roots *)
dp = Function[{f, g}, Integrate[f * g * Exp[-x^2], {x, -Infinity, Infinity}]]
nm = Function[f, Sqrt[dp[f, f]]]
```

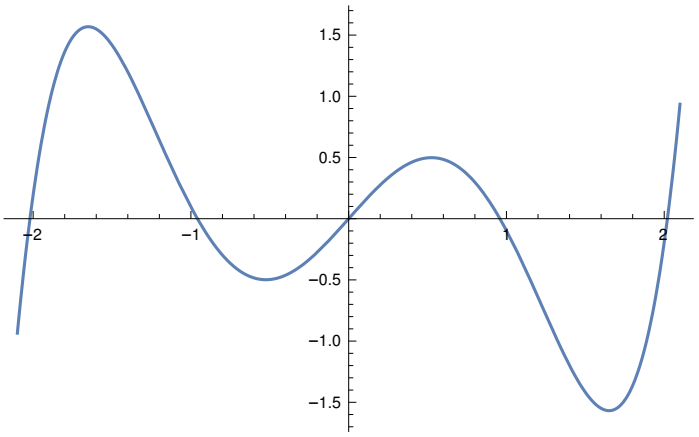
Out[1]= $\text{Function}\left[\{f, g\}, \int_{-\infty}^{\infty} f\, g\, \text{Exp}\left[-x^2\right] dx\right]$

Out[2]= $\text{Function}\left[f, \sqrt{\text{dp}[f, f]}\right]$

In[3]:=
```
N0 = 5;
For[k = 0, k <= N0, k++,
  w[k] = x^k;
  For[j = 0, j < k, j++,
    w[k] = w[k] - dp[H[j], x^k] * H[j]];
  H[k] = Simplify[w[k] / nm[w[k]]]]
```

In[5]:=
```
Plot[H[5], {x, -2.1, 2.1}]
(* next compute the roots of H[5] so we don't have
   to type them in ourselves *)
X = x /. Solve[H[5] == 0, x]
Xn = N[X, 16];
Print[Xn]
```

Out[5]=



Out[6]= $\left\{0, -\sqrt{\frac{5}{2}+\sqrt{\frac{5}{2}}}, \sqrt{\frac{5}{2}+\sqrt{\frac{5}{2}}}, -\sqrt{\frac{1}{2}\left(5-\sqrt{10}\right)}, \sqrt{\frac{1}{2}\left(5-\sqrt{10}\right)}\right\}$

```
{0, -2.020182870456086, 2.020182870456086, -0.9585724646138185, 0.9585724646138185}
```

In[9]:=
```
For[j = 0, j < 5, j++,
 B[j] = dp[x^j, 1];
 Bn[j] = N[B[j], 16]]
Print[Table[Bn[j], {j, 0, 4}]]
```

```
{1.772453850905516, 0, 0.8862269254527580, 0, 1.329340388179137}
```

In[11]:=
```
(* We could put the roots Xn and the integrals Bn into
   a Julia program to calculate the weights Wn, but
   it is easier to do it here. *)
EQ[0] = Bn[0] == Sum[W[k], {k, 1, 5}];
For[j = 1, j < 5, j++,
 EQ[j] = Bn[j] == Sum[W[k] * Xn[[k]]^j, {k, 1, 5}]]
```

In[13]:=
```
Vars = Table[W[j], {j, 1, 5}]
Eqns = Table[EQ[j], {j, 0, 4}]
```

Out[13]=
```
{W[1], W[2], W[3], W[4], W[5]}
```

Out[14]=
```
{1.772453850905516 == W[1] + W[2] + W[3] + W[4] + W[5],
 0 == -2.020182870456086 W[2] + 2.020182870456086 W[3] -
   0.9585724646138185 W[4] + 0.9585724646138185 W[5],
 0.8862269254527580 == 4.081138830084190 W[2] + 4.081138830084190 W[3] +
   0.918861169915810 W[4] + 0.918861169915810 W[5],
 0 == -8.24464675648927 W[2] + 8.24464675648927 W[3] - 0.880795016284135 W[4] +
   0.880795016284135 W[5], 1.329340388179137 == 16.65569415042095 W[2] +
   16.65569415042095 W[3] + 0.844305849579052 W[4] + 0.844305849579052 W[5]}
```

In[15]:=
```
(* since the Wn have to be paired with the Xn
   make a table to show which goes with which *)Wn = Flatten[Vars /. Solve[Eqns, Vars]]
```

Out[15]=
```
{0.94530872048294, 0.0199532420590459,
 0.0199532420590459, 0.393619323152241, 0.393619323152241}
```

In[16]:=
```
T1 = Table[{Xn[[j]], Wn[[j]]}, {j, 1, 5}];
Grid[Prepend[T1, {"Xn", "Wn"}]]
```

Out[17]=

Xn	Wn
0	0.94530872048294
-2.020182870456086	0.0199532420590459
2.020182870456086	0.0199532420590459
-0.9585724646138185	0.393619323152241
0.9585724646138185	0.393619323152241