In[1]:=
```
(* Question 3 *)
eq = y ' -> Function[t, f[t, y[t]]]
k1 = f[tn, y[tn]]
k2 = f[tn + h / 3, y[tn] + h * k1 / 3]
k3 = f[tn + 2 * h / 3, y[tn] + 2 * h * k2 / 3]
ynp1 = y[tn] + h * (k1 + 3 * k3) / 4
tau = y[tn + h] - ynp1
```

Out[1]= y′ → Function[t, f[t, y[t]]]

Out[2]= f[tn, y[tn]]

Out[3]= $f\left[\frac{h}{3} + tn,\ \frac{1}{3} h\, f[tn, y[tn]] + y[tn]\right]$

Out[4]= $f\left[\frac{2h}{3} + tn,\ \frac{2}{3} h\, f\left[\frac{h}{3} + tn,\ \frac{1}{3} h\, f[tn, y[tn]] + y[tn]\right] + y[tn]\right]$

Out[5]= $\frac{1}{4} h \left(f[tn, y[tn]] + 3 f\left[\frac{2h}{3} + tn,\ \frac{2}{3} h\, f\left[\frac{h}{3} + tn,\ \frac{1}{3} h\, f[tn, y[tn]] + y[tn]\right] + y[tn]\right]\right) + y[tn]$

Out[6]= $-\frac{1}{4} h \left(f[tn, y[tn]] + 3 f\left[\frac{2h}{3} + tn,\ \frac{2}{3} h\, f\left[\frac{h}{3} + tn,\ \frac{1}{3} h\, f[tn, y[tn]] + y[tn]\right] + y[tn]\right]\right) - y[tn] + y[h + tn]$

In[7]:=
```
(* check the method is consistent *)
tau /. h -> 0
```

Out[7]= 0

In[8]:=
```
(* now take derivatives until the corresponding
   terms in the Taylor series cease to vanish *)
dj = tau;
For[j = 1, j < 6, j ++,
 dj = D[dj, h] /. eq;
 djh0 = Simplify[dj /. h → 0];
 Print["d", j, "h0=", djh0];
 If[djh0 =!= 0,
  Print["truncation error is O(h^", j, ")"];
  Print["order of method is O(h^", j - 1, ")"];
  Break[]]]
```

d1h0=0

d2h0=0

d3h0=0

$$d4h0=\frac{1}{9}\Big(f[tn, y[tn]]^3 f^{(0,3)}[tn, y[tn]] + 9 f^{(0,1)}[tn, y[tn]]^2 f^{(1,0)}[tn, y[tn]] + 3 f^{(1,0)}[tn, y[tn]] f^{(1,1)}[tn, y[tn]] + 3 f[tn, y[tn]]^2 \big(2 f^{(0,1)}[tn, y[tn]] f^{(0,2)}[tn, y[tn]] + f^{(1,2)}[tn, y[tn]]\big) + 3 f^{(0,1)}[tn, y[tn]] f^{(2,0)}[tn, y[tn]] + 3 f[tn, y[tn]] \big(3 f^{(0,1)}[tn, y[tn]]^3 + f^{(0,2)}[tn, y[tn]] f^{(1,0)}[tn, y[tn]] + 3 f^{(0,1)}[tn, y[tn]] f^{(1,1)}[tn, y[tn]] + f^{(2,1)}[tn, y[tn]]\big) + f^{(3,0)}[tn, y[tn]]\Big)$$

truncation error is O(h^4)

order of method is O(h^3)