

In[1]:= **dp = Function[{f, g}, Integrate[f \* g, {x, -1, 1}]]**

Out[1]=  $\text{Function}\left[\{f, g\}, \int_{-1}^1 f g dx\right]$

In[2]:= **nm = Function[f, Sqrt[dp[f, f]]]**

Out[2]=  $\text{Function}\left[f, \sqrt{\text{dp}[f, f]}\right]$

In[3]:= **dp[x^2, x^4]**

Out[3]=  $\frac{2}{7}$

In[4]:= **nm[x]**

Out[4]=  $\sqrt{\frac{2}{3}}$

In[5]:= **w[0] = 1**

Out[5]= 1

In[6]:= **v[0] = w[0] / nm[w[0]]**

Out[6]=  $\frac{1}{\sqrt{2}}$

In[7]:= **w[1] = x - dp[v[0], x] \* v[0]**

Out[7]= x

In[8]:= **v[1] = w[1] / nm[w[1]]**

Out[8]=  $\sqrt{\frac{3}{2}} x$

In[9]:= **w[2] = x^2 - dp[v[0], x^2] \* v[0] - dp[v[1], x^2] \* v[1]**

Out[9]=  $-\frac{1}{3} + x^2$

In[10]:= **v[2] = Expand[w[2] / nm[w[2]]]**

Out[10]=  $-\frac{\sqrt{\frac{5}{2}}}{2} + \frac{3}{2} \sqrt{\frac{5}{2}} x^2$

In[11]:= **N0 = 8**

Out[11]= 8

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In[12]:= For[k = 0, k ≤ N0, k ++,
  w[k] = x ^ k;
  For[j = 0, j < k, j ++,
    w[k] = w[k] - dp[v[j], x ^ k] * v[j]];
  v[k] = Simplify[w[k] / nm[w[k]]];
  Print["v[" , k, "]=", v[k]]]

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$$v[0] = \frac{1}{\sqrt{2}}$$

$$v[1] = \sqrt{\frac{3}{2}} x$$

$$v[2] = \frac{1}{2} \sqrt{\frac{5}{2}} (-1 + 3x^2)$$

$$v[3] = \frac{1}{2} \sqrt{\frac{7}{2}} x (-3 + 5x^2)$$

$$v[4] = \frac{3(3 - 30x^2 + 35x^4)}{8\sqrt{2}}$$

$$v[5] = \frac{1}{8} \sqrt{\frac{11}{2}} x (15 - 70x^2 + 63x^4)$$

$$v[6] = \frac{1}{16} \sqrt{\frac{13}{2}} (-5 + 105x^2 - 315x^4 + 231x^6)$$

$$v[7] = \frac{1}{16} \sqrt{\frac{15}{2}} x (-35 + 315x^2 - 693x^4 + 429x^6)$$

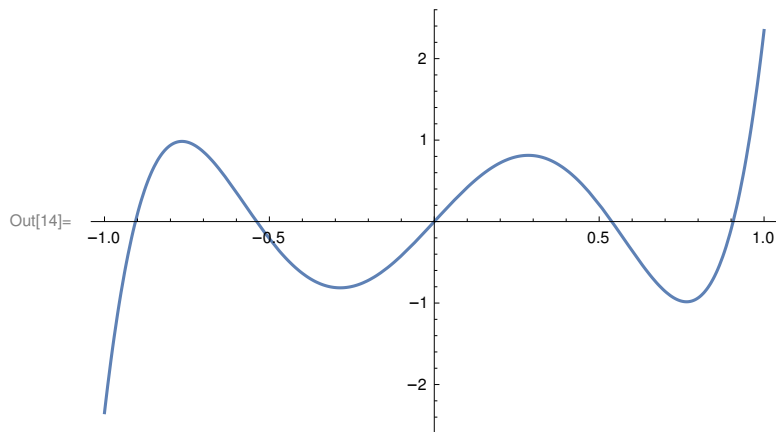
$$v[8] = \frac{1}{128} \sqrt{\frac{17}{2}} (35 - 1260x^2 + 6930x^4 - 12012x^6 + 6435x^8)$$

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In[13]:=

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In[14]:= `Plot[v[5], {x, -1, 1}]`



In[15]:= `R5 = x /. Solve[v[5] == 0, x]`

Out[15]=  $\left\{ 0, -\frac{1}{3} \sqrt{\frac{1}{7} (35 - 2\sqrt{70})}, \frac{1}{3} \sqrt{\frac{1}{7} (35 - 2\sqrt{70})}, -\frac{1}{3} \sqrt{\frac{1}{7} (35 + 2\sqrt{70})}, \frac{1}{3} \sqrt{\frac{1}{7} (35 + 2\sqrt{70})} \right\}$

In[16]:= `R5n = N[R5, 16]`

Out[16]=  $\{0, -0.5384693101056831, 0.5384693101056831, -0.9061798459386640, 0.9061798459386640\}$