In[1]:=
```
eq = y ' -> Function[s, f[s, y[s]]]
```

Out[1]=
```
y' → Function[s, f[s, y[s]]]
```

In[2]:=
```
yp = y[xn - h] + 2 * h * f[xn, y[xn]]
```

Out[2]=
```
2 h f[xn, y[xn]] + y[-h + xn]
```

In[3]:=
```
ynp1 = y[xn] + h / 2 * (f[xn, y[xn]] + f[xn + h, yp])
```

Out[3]=
$$\frac{1}{2} h \left(f[xn, y[xn]] + f[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]]\right) + y[xn]$$

In[4]:=
```
r = y[xn + h] - ynp1
```

Out[4]=
$$-\frac{1}{2} h \left(f[xn, y[xn]] + f[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]]\right) - y[xn] + y[h + xn]$$

In[5]:=
```
r /. h → 0
```

Out[5]=
```
0
```

In[6]:=
```
T1 = D[r, h]
```

Out[6]=
$$\frac{1}{2} \left(-f[xn, y[xn]] - f[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]]\right) + y'[h + xn] -$$
$$\frac{1}{2} h \left((2 f[xn, y[xn]] - y'[-h + xn]) f^{(0,1)}[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]] + f^{(1,0)}[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]]\right)$$

In[7]:=
```
dr = T1 /. eq
```

Out[7]=
$$\frac{1}{2} \left(-f[xn, y[xn]] - f[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]]\right) + f[h + xn, y[h + xn]] -$$
$$\frac{1}{2} h \left((2 f[xn, y[xn]] - f[-h + xn, y[-h + xn]]) f^{(0,1)}[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]] + f^{(1,0)}[h + xn, 2 h f[xn, y[xn]] + y[-h + xn]]\right)$$

In[8]:=
```
dr /. h → 0
```

Out[8]=
```
0
```

In[9]:=
```
ddr = D[dr, h] /. eq
```

Out[9]=
$$-(2\,f[xn,\,y[xn]]-f[-h+xn,\,y[-h+xn]])\,f^{(0,1)}[h+xn,\,2\,h\,f[xn,\,y[xn]]+y[-h+xn]]+$$
$$f[h+xn,\,y[h+xn]]\,f^{(0,1)}[h+xn,\,y[h+xn]]-f^{(1,0)}[h+xn,\,2\,h\,f[xn,\,y[xn]]+y[-h+xn]]+$$
$$f^{(1,0)}[h+xn,\,y[h+xn]]-\frac{1}{2}\,h\left(f^{(0,1)}[h+xn,\,2\,h\,f[xn,\,y[xn]]+y[-h+xn]]\right.$$
$$\left(f[-h+xn,\,y[-h+xn]]\,f^{(0,1)}[-h+xn,\,y[-h+xn]]+f^{(1,0)}[-h+xn,\,y[-h+xn]]\right)+$$
$$(2\,f[xn,\,y[xn]]-f[-h+xn,\,y[-h+xn]])\,f^{(1,1)}[h+xn,\,2\,h\,f[xn,\,y[xn]]+y[-h+xn]]+$$
$$(2\,f[xn,\,y[xn]]-f[-h+xn,\,y[-h+xn]])$$
$$\left((2\,f[xn,\,y[xn]]-f[-h+xn,\,y[-h+xn]])\,f^{(0,2)}[h+xn,\,2\,h\,f[xn,\,y[xn]]+y[-h+xn]]+\right.$$
$$\left.\left.f^{(1,1)}[h+xn,\,2\,h\,f[xn,\,y[xn]]+y[-h+xn]]\right)+f^{(2,0)}[h+xn,\,2\,h\,f[xn,\,y[xn]]+y[-h+xn]]\right)$$

In[10]:=
```
ddr /. h → 0
```

Out[10]=
```
0
```

In[11]:=
```
d3r = D[ddr, h] /. eq;
```

In[12]:=
```
d3r /. h → 0
```

Out[12]=
$$\frac{1}{2}\left(-f^{(0,1)}[xn,\,y[xn]]\left(f[xn,\,y[xn]]\,f^{(0,1)}[xn,\,y[xn]]+f^{(1,0)}[xn,\,y[xn]]\right)-f[xn,\,y[xn]]\,f^{(1,1)}[xn,\,y[xn]]-\right.$$
$$\left.f[xn,\,y[xn]]\left(f[xn,\,y[xn]]\,f^{(0,2)}[xn,\,y[xn]]+f^{(1,1)}[xn,\,y[xn]]\right)-f^{(2,0)}[xn,\,y[xn]]\right)$$