Last section of 2.5

Backwards differentiation formulas

Last time we used the theorem

$$\sigma(w) = \frac{\rho(w)}{\log w} + O(|w-1|^p)$$

to find σ given a ρ the satisfies the root condition. This guarantees the method converges and then taking $p=s$ for an explicit method guarantees optimal order (or $p=s+1$ for an implicit method is also good).

Backwards differentiation formula solves for $\rho(w)$ given $\sigma(w)$:

$$\rho(w) = \log w \, \sigma(w) + O(|w-1|^{p+1})$$

when $\sigma(w) = \beta w^s$.

some constant...

Note this is an implicit method since degree of σ is s and the same as ρ.

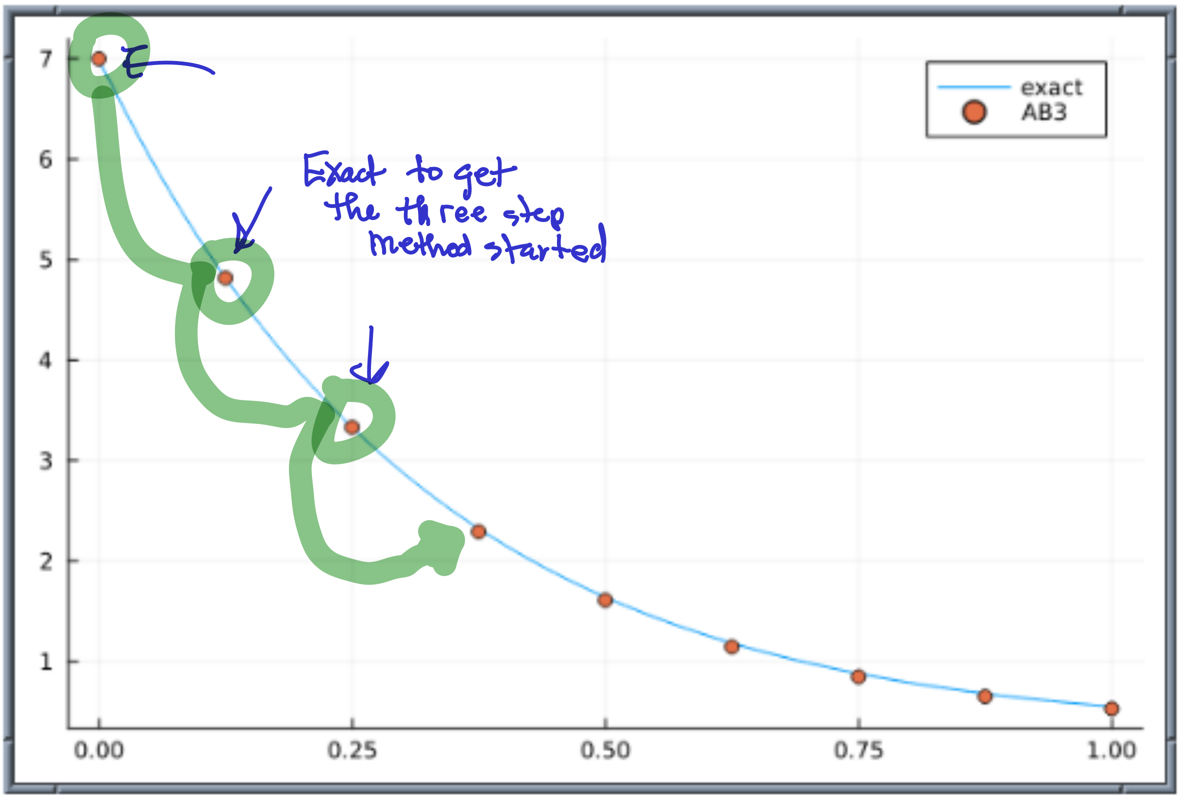
$$\sum_{m=0}^{s} a_m y_{n+m} = h \sum_{m=0}^{s} \beta_m f(t_{n+m}, y_{n+m}) \quad \text{where} \quad n = 0, 1, \ldots.$$

BDF

$$\sum_{m=0}^{s} a_m y_{n+m} = h\beta f(t_{n+s}, y_{n+s})$$

implicit on the RHS this increases stability for making rough approximations.

(to be discussed later)



w^(s-2)*(w^2-1)

```
julia> rho(w)=w^(s-1)*(w-1)
rho (generic function with 1 method)
```

← define ρ for AB method

```
julia> t=Taylor1(s)
 1.0 t + O(t⁴)
```

← Taylor series in t as s degree polynomials

$w = \xi + 1$

```
julia> sigma=rho(t+1)/log(t+1)
 1.0 + 2.5 t + 1.9166666666666667 t² + O(t³)
```

← Create Taylor series expanded in powers of ξ where $\xi = w-1$.

note its now $O(|w-1|^3)$

```
julia> z=evaluate(sigma,[t-1])[1]
 0.4166666666666674 - 1.3333333333333335 t
 + 1.9166666666666667 t² + O(t³)
```

shift back to the w variable..

```
julia> b=z.coeffs
3-element Vector{Float64}:
  0.4166666666666674
 -1.3333333333333335
  1.9166666666666667
```

$b_0 = b[1]$
$b_1 = b[2]$
$b_2 = b[3]$