Required Texts:
Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, Fifth Edition by Richard Haberman

A partial differential equation is an equation that involves partial derivatives...

example →
$$\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = 0.$$

Mathematical approach: Think about a PDE and try to understand it if is well posed; That is
1. There is a solution.
2. The solution is unique.
3. How do the solutions behave?

Physical Approach; Study a physical problem. write down some differential equations describing the physics;
Assume that there is a unique solution because its a physical problem, and we already have intuition about the behavior of the solution....

Historically: Physical approach... still used...

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Get started with Heat Conduction

Modeling: Idea: Heat energy.
To understand the distribution of heat in a material we define energy density.

Energy density = Energy / unit of volume.

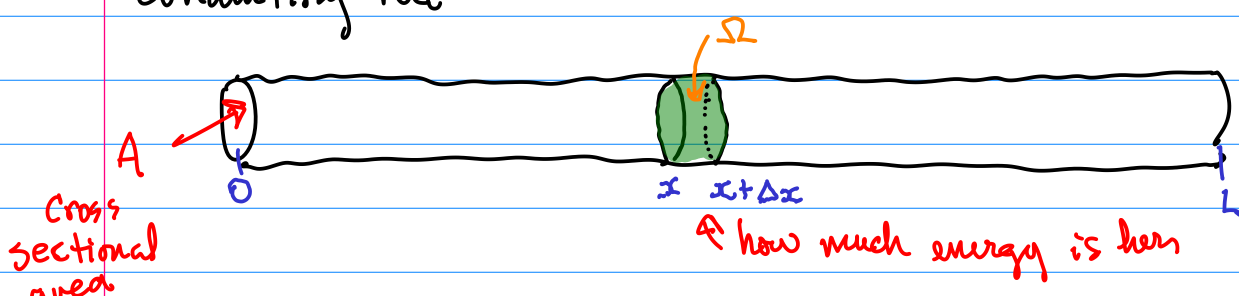
Energy: measure in calories, Joules, BTUs...

Volume: measure in $cm^3$, $ft^3$, $m^3$.

| | | |
|---|---|---|
| $[E]$ | units of energy | Joules, BTUs |
| $[L]^3 = [V]$ | units of volume | $m^3$ |
| $[L]$ | unit of length | m |
| $[T]$ | unit of time | seconds |
| $[L]^2 = [A]$ | units of Area | $m^2$ |
| $[M]$ | units of mass | grams |
| $[u]$ | units of temperature | $^\circ C$ |

Example of a heat conduction problem

conducting rod




$$\text{Energy} = \int_\Omega e \, dV \approx A \int_x^{x+\Delta x} e(s,t)\, ds$$

energy density

assuming the density is constant across the cross sectional area.

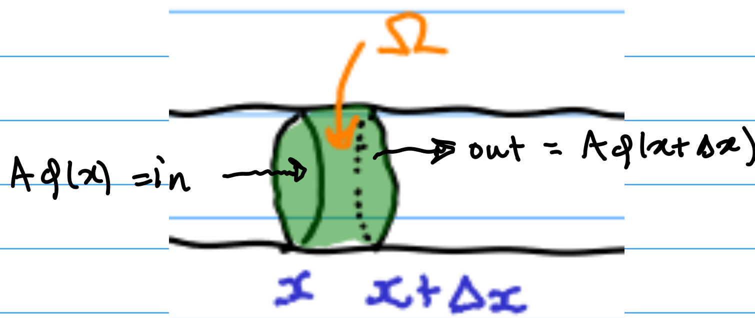
Conservation of energy.

| | | | | |
|---|---|---|---|---|
| rate of change of heat energy in time | = | heat energy flowing across boundaries per unit time | + | heat energy generated inside per unit time. |

Heat flowing across the boundaries

Heat flux : $\varphi$ unit $\dfrac{[E]}{[T][A]} = \dfrac{[E]}{[T][L]^2}$

Total heat flux through an area $= \int \varphi \, dA = A\varphi(x)$




rate of change in heat energy.
through the surface: $A\varphi(x) - A\varphi(x+\Delta x)$

Heat generated inside the region

Heat source: $Q$ units $\dfrac{[E]}{[T][V]} = \dfrac{[E]}{[T][L]^3}$

rate of heat generated inside.

$$\int_\Omega Q \, dV = A \int_x^{x+\Delta x} Q(s,t)\, ds$$

at time $t$

at location $s$.

$$\frac{\partial}{\partial t} \underbrace{\cancel{A}\int_x^{x+\Delta x} e(\overset{\text{locations}}{s},\overset{\text{time } t}{t})\,ds}_{\text{energy in } \Omega} = \cancel{A}\phi(x) - \cancel{A}\phi(x+\Delta x) + \cancel{A}\int_x^{x+\Delta x} Q(s,t)\,ds$$

Turn this into a differential equation by taking $\Delta x \to 0$.

$$\frac{\partial}{\partial t} \underbrace{\cancel{\frac{1}{\Delta x}}\cancel{\int_x^{x+\Delta x}} e(s,\overset{\text{time}}{t})\,ds}_{\substack{\text{average value of } e \\ \text{over a smaller} \\ \text{and smaller interval,}}} = \underbrace{\frac{\phi(x,t) - \phi(x+\Delta x,t)}{\Delta x}}_{\substack{\text{negative the} \\ \text{definition} \\ \text{of} \\ \text{derivative}}} + \cancel{\frac{1}{\Delta x}}\cancel{\int_x^{x+\Delta x}} Q(s,t)\,ds$$

The partial differential equation is:

$$\frac{\partial}{\partial t} e(x,t) = -\frac{\partial \phi(x,t)}{\partial x} + Q(x,t)$$