$$\frac{\partial}{\partial t} e(x,t) = -\frac{\partial \phi(x,t)}{\partial x} + Q(x,t)$$

time

energy density

energy flux passing through the surface

rate of energy production per unit volume

$$e \sim \frac{[E]}{[L]^3} \qquad \phi \sim \frac{[E]}{[T][L]^2} \qquad Q \sim \frac{[E]}{[T][L]^3}$$

$$\frac{\partial e}{\partial t} \sim \frac{[E]}{[T][L]^3} \qquad \frac{\partial \phi}{\partial x} \sim \frac{[E]}{[T][L]^3}$$

all have the same physical dimensions ☑

What is energy? Relationship between thermal energy and temperature.

Calories —— The amount of energy need to raise 1 gram of water by 1 degree celsius.

Joule — The amount of energy needed to move 1 coulomb of electrons through an electric potential of 1 volt.

↳ in terms of heat

The amount of energy need to raise 0.239 grams of water by 1 degree celsius.

approximate conversion factor

BTU — The amount of energy to raise 1 pound of water by 1 degree Fahrenheit.

$$1\text{ BTU} = 1\text{ pound }{}^\circ F \cdot \frac{5}{9}\frac{C^\circ}{F^\circ}\, 453.6\frac{\text{grams}}{\text{pound}} = 252\text{ calories}$$

$$F = \frac{9}{5}C + 32$$

```
julia> 1*5/9*453.6
252.00000000000003
```

$$\approx 252\text{ calories}\,\frac{1}{0.239}\,\frac{\text{Joules}}{\text{Calorie}} = 1054\text{ Joules}$$

```
julia> 252/0.239
1054.3933054393306
```

Goal: Rewrite energies in terms of temperatures.

Heat capacity: $c(x) \sim$ amount of energy per unit mass per degree $\sim \dfrac{[E]}{[M][u]}$

units of mass

units of temp.

What is the heat capacity of water?

$$c(x) = 1\,\frac{\text{Calories}}{\text{gram}\cdot{}^\circ C} = 1\,\frac{\text{BTU}}{\text{pound}\cdot{}^\circ F} = 4.184\,\frac{\text{Joules}}{\text{grams}\cdot{}^\circ C}$$

maybe the material used to make the conducting rod changes with position...

```
julia> 1/0.239
4.184100418410042
```

Density $\rho(x) \sim \frac{[M]}{[L]^3}$

Try to express energy density in terms of these other quantities:

recall
$c(x) \sim \frac{[E]}{[M][u]}$

$$e \sim \frac{[E]}{[L]^3} = \underbrace{\frac{[E]}{[M][u]}}_{\text{heat capacity}} \underbrace{\frac{[M]}{[L]^3}}_{\text{density}} \underbrace{[u]}_{\text{temperature.}}$$

$$e(x,t) = c(x)\,\rho(x)\,u(x,t)$$ by definition of c, ρ, u.

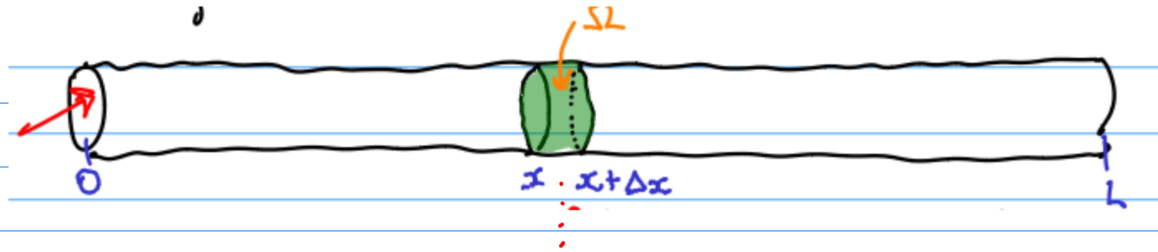
Substitute

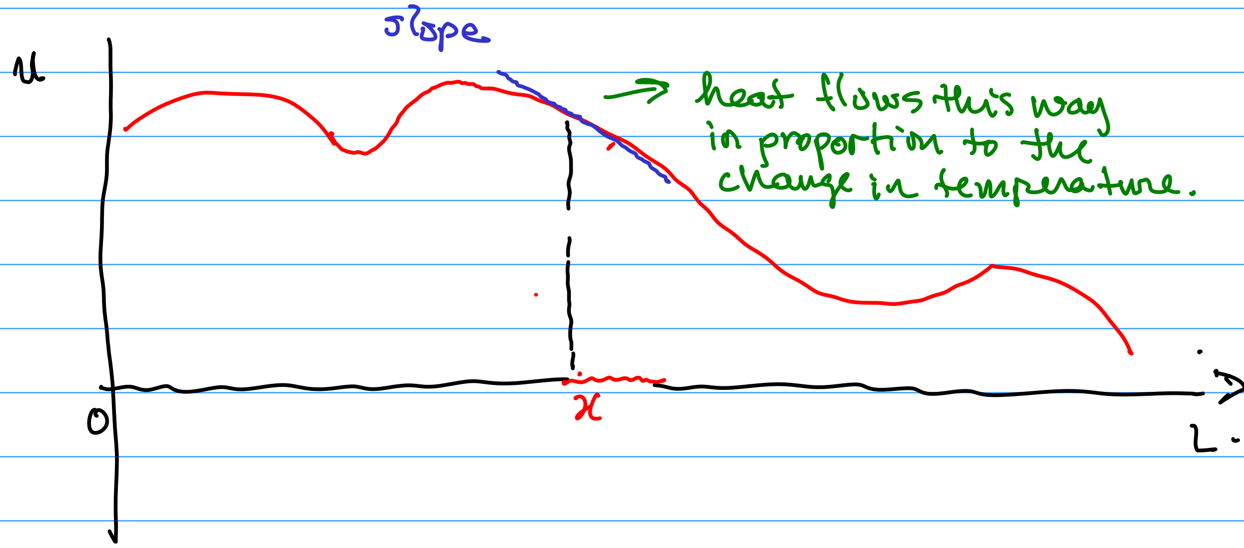
$$\boxed{\frac{\partial}{\partial t} e(x,t) = -\frac{\partial \phi(x,t)}{\partial x} + Q(x,t)}$$

$$c(x)\,\rho(x)\,\frac{\partial u(x,t)}{\partial t} = -\frac{\partial \phi(x,t)}{\partial x} + Q(x,t).$$

Now let's relate the flux ϕ to the temperature.

Need physics for this: Fourier's Law.





$$\phi(x,t) = -K_0(x)\,\frac{\partial u}{\partial x}$$

$$c(x)\,\rho(x)\,\frac{\partial u(x,t)}{\partial t} = -\frac{\partial \phi(x,t)}{\partial x} + Q(x,t)$$

$$c(x)\,\rho(x)\,\frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K_0(x)\,\frac{\partial u}{\partial x}\right) + Q(x,t)$$

Simple example: Suppose no heat is being produced in the conducting bar so $Q(x,t) = 0$.

Also suppose the conductivity is constant throughout the bar. $K_0(x) = K_0$ independent of x.

$$c(x)\,\rho(x)\,\frac{\partial u(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K_0(x)\,\frac{\partial u}{\partial x} \right) + Q(x,t)$$

Heat equation that we study:

$$c(x)\,\rho(x)\,\frac{\partial u(x,t)}{\partial t} = K_0\,\frac{\partial^2 u(x,t)}{\partial x^2}$$