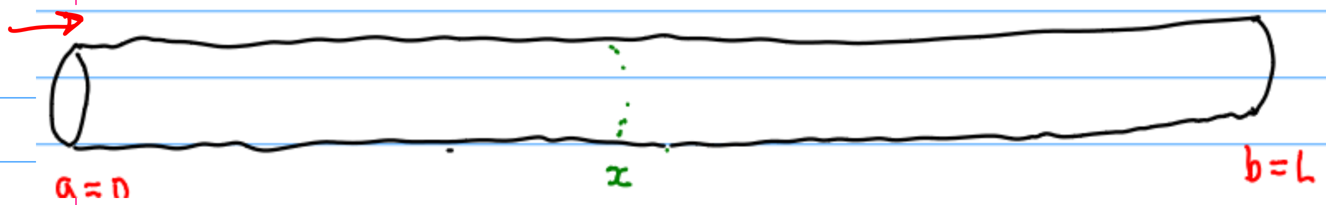



$\phi(0,t)=0$ for $t \geq 0$ $\qquad$ $\phi(L,t)=0$ for $t\geq 0$

Insulating boundary conditions

Energy flux $\rightarrow$

Fourier's law

$$\boxed{\phi = -K_0(x)\frac{\partial u}{\partial x}}$$

$$-K_0(0)\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0 \qquad \text{assuming } K_0(0)\neq 0,\ K_0(L)\neq 0. \qquad -K_0(L)\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

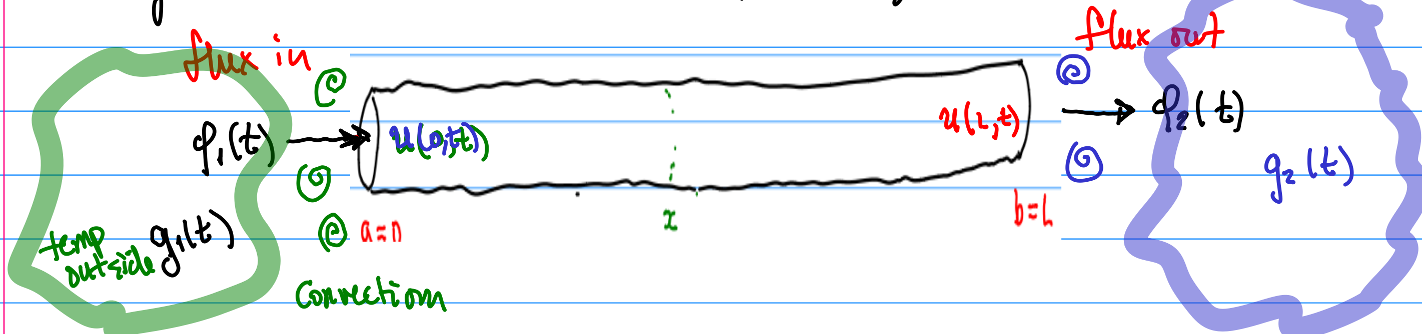
Prescrible the flux at the endpoints

$$-K_0(0)\left.\frac{\partial u}{\partial x}\right|_{x=0} = \phi_1(t) \qquad -K_0(L)\left.\frac{\partial u}{\partial x}\right|_{x=L} = \phi_2(t)$$

equivalently.

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = h_1(t) \qquad \left.\frac{\partial u}{\partial x}\right|_{x=L} = h_2(t)$$

Boundary conditions on Newton's law of cooling...



$$\phi(0,t) = H_1\left(g_1(t) - u(0,t)\right) \qquad \phi(L,t) = H_2\left(u(L,t) - g_2(t)\right)$$

$$-K_0(0)\left.\frac{\partial u}{\partial x}\right|_{x=0} = H_1\left(g_1(t) - u(0,t)\right) \qquad -K_0(L)\left.\frac{\partial u}{\partial x}\right|_{x=L} = H_2\left(u(L,t) - g_2(t)\right)$$

**Newton's law of cooling.** When a one-dimensional rod is in contact at the boundary with a moving fluid (e.g., air), then neither the prescribed temperature nor the prescribed heat flow may be appropriate. For example, let us imagine a very warm rod in contact with cooler moving air. Heat will leave the rod, heating up the air. The air will then carry the heat away. This process of heat transfer is called **convection**. However, the air will be hotter near the rod. Again, this is a complicated problem; the air temperature will actually vary with distance from the rod (ranging between the bath and rod temperatures). Experiments show that, as a good approximation, the heat flow leaving the rod is proportional to the temperature difference between the bar and the prescribed external temperature. This boundary condition is called **Newton's law of cooling**. If it is valid at $x = 0$, then

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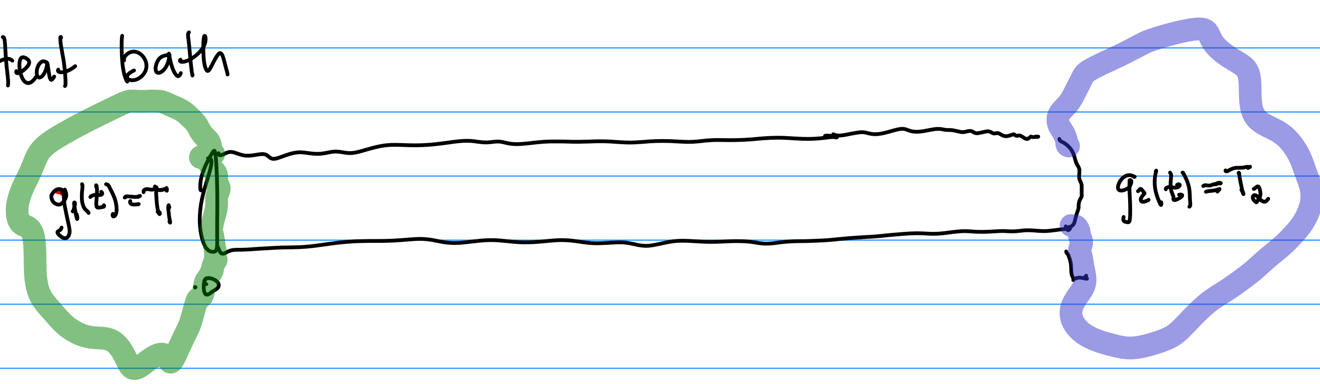
<u>Idea</u> – predict the future as $t \to \infty$.

Since intuitively heat diffuses with the warm getting cooler while cool heats up, we expect the solution to tend to a time independant state as $t \to \infty$ provided the boundary conditions don't depend on time...

<u>Note</u>: need to be carefull with boundary conditions to make sure an equilibrium state even exists

Heat bath



Simple heat equation $K_0 = \text{const.}$ $\rho = \text{const.}$ $c = \text{const}$

conductivity density heat capacity.

$$k = \frac{K_0}{\rho c}$$

PDE. $$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

initial cond. $u(x,0) = f(x)$ for $x \in [0,L]$

boundary cond. $u(0,t) = T_1$ $u(L,t) = T_2$

Equilibrium: so suppose $u$ does not depend on $t$.

ODE $$\frac{d^2u}{dx^2} = 0, \quad u(0) = T_1, \quad u(L) = T_2$$

$$\frac{du}{dx} = \int \frac{d^2u}{dx^2}\,dx = \int 0\,dx = C$$

$$u = \int \frac{du}{dx}\,dx = \int C\,dx = Cx + D$$

$$u(0) = C \cdot 0 + D = D = T_1$$

$$u(L) = C \cdot L + T_1 = T_2 \qquad C = (T_2 - T_1)/L$$

$$u(x) = \frac{T_2 - T_1}{L} x + T_1$$

Insulated boundary equilibrium state

PDE. $\cancel{\frac{\partial u}{\partial t}} = k\frac{\partial^2 u}{\partial x^2}$

initial cond. $u(x,0) = f(x)$ for $x \in [0,L]$

boundary cond. $\frac{\partial u}{\partial x}(0,t) = 0$ $\frac{\partial u}{\partial x}(L,t) = 0$

ODE $\frac{d^2 u}{dx^2} = 0$ , $u'(0) = 0$ , $u'(L) = 0$

$$\frac{du}{dx} = \int \frac{d^2 u}{dx^2}\,dx = \int 0\,dx = C$$

$$u = \int \frac{du}{dx}\,dx = \int C\,dx = Cx + D$$

$u'(0) = C = 0$

$u'(L) = C = 0$ what is D ?

Since insulated then energy conservation means the energy in the initial state is the same as the equilibrium state