Text for the class

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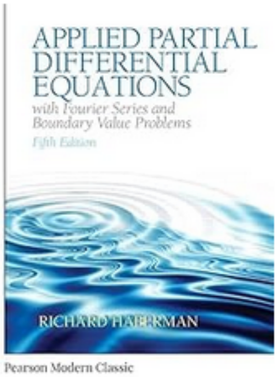


Applied Partial Differential Equations with Fourier Series and Boundary Value Problems (Classic Version) (Pearson Modern Classics for Advanced Mathematics Series)

5th Edition

by Richard Haberman (Author)

4.3 ★★★★☆ 107 ratings   See all formats and editions

Review of linear ODEs.

Form:

$$y' + p(x)y = f(x)$$

$$\underbrace{y' + 3y}_{} = \sin x$$

turn this into something that comes from the product rule...

$$\frac{d\,e^{3x}}{dx} = 3e^{3x}$$

$$\underbrace{y'e^{3x} + 3e^{3x}y}_{} = e^{3x}\sin x$$

$$\frac{d\,ye^{3x}}{dx} = y'e^{3x} + y\,3e^{3x}$$

$$\frac{d\,ye^{3x}}{dx} = e^{3x}\sin x$$

Now integrate both sides

$$\int_{x_0}^{x} \frac{d\, y(s)e^{3s}}{ds}\, ds = \int_{x_0}^{x} e^{3s}\sin s \, ds$$

Thus,

$$y(x)e^{3x} - y(x_0)e^{3x_0} = \int_{x_0}^{x} e^{3s}\sin s\, ds$$

$$y(x) = y(x_0)\, e^{3(x_0 - x)} + e^{-3x}\int_{x_0}^{x} e^{3s}\sin s\, ds.$$

$$y(x) = y(x_0)\, e^{3(x_0 - x)} + \int_{x_0}^{x} e^{3(s-x)}\sin s\, ds.$$

Same thing

$$y' + p(x)y = f(x) \qquad \mu(x) = e^{\int_{x_0}^{x} p(s)ds}$$

$$\frac{d\mu(x)}{dx} = e^{\int_{x_0}^{x} p(s)ds}\, \frac{d}{dx}\int_{x_0}^{x} p(s)ds = \mu(x)p(x)$$

$$\underbrace{y'\mu(x) + \mu(x)p(x)y}_{} = \mu(x)f(x)$$

$$\frac{d}{dx}\left(y(x)\mu(x)\right) = y'(x)\mu(x) + y(x)\mu'(x)$$

$$= y'(x)\mu(x) + y(x)\mu(x)p(x)$$

$$\frac{d}{dx}\left(y(x)\mu(x)\right) = \mu(x)f(x).$$

$$\int_{x_0}^{x} \frac{d}{ds}\big(y(s)\mu(s)\big)\,ds = \int_{x_0}^{x} \mu(s) f(s)\,ds$$

$$y(x)\mu(x) - y(x_0)\mu(x_0) = \int_{x_0}^{x} \mu(s) f(s)\,ds$$

$$y(x) = y(x_0)\frac{\mu(x_0)}{\mu(x)} + \frac{1}{\mu(x)}\int_{x_0}^{x} \mu(s) f(s)\,ds$$

$$\mu(x) = e^{\int_{x_0}^{x} p(z)\,dz}$$

$$\frac{\mu(x_0)}{\mu(x)} = \frac{e^{\int_{x_0}^{x_0} p(z)\,dz}}{e^{\int_{x_0}^{x} p(z)\,dz}} = e^{-\int_{x_0}^{x} p(z)\,dz}$$

$$y(x) = e^{-\int_{x_0}^{x} p(z)\,dz}\left(y(x_0) + \int_{x_0}^{x} e^{\int_{x_0}^{s} p(z)\,dz} f(s)\,ds\right)$$

$$y(x) = e^{-\int_{x_0}^{x} p(z)\,dz}\quad y(x_0) + \int_{x_0}^{x} e^{\int_{x}^{s} p(z)\,dz} f(s)\,ds$$

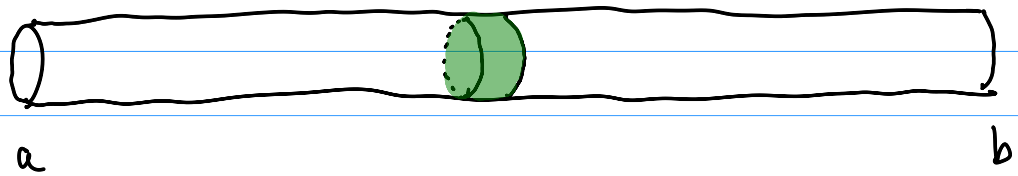
$$\boxed{e^{-\int_{x_0}^{x} p(z)\,dz}\, e^{\int_{x_0}^{s} p(z)\,dz} = e^{-\int_{x_0}^{x} p(z)\,dz + \int_{x_0}^{s} p(z)\,dz}}$$

Partial differential equation: literally an equation involving partial derivatives.

Example:

$$\frac{\partial u}{\partial t} + 3\frac{\partial u}{\partial x} = 0$$

Physics: Heat conduction in a 1-dim rod.



So thin the heat can be represented uniform in the radial direction