Conservation of energy to derive a PDE.

What is heat energy???

Things we know

units	type	measured as
$[L]$	length	m meters ft feet cm centimeters
$[A]=[L]^2$	area	m^2 square meters
$[V]=[L]^3$	volume	m^3 cubic meters
$[T]$	time	s seconds min minutes. hr hours.
$[M]$	mass	g grams lb pounds
$[u]$	temperature	°C celcius °F Fahrenheit... °K Kelvin
$[E]$	energy	J Joules BTUs British thermal units calories Watt·hours..

More about energy:

Calories energy needed to raise 1 gram of water by 1 degree Celcius (or Kelvin)

BTU — energy needed to raise 1 pound of water by 1 degree Fahrenheit (or Kelvin)

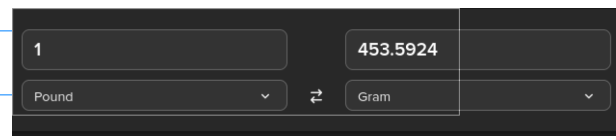
Joules — energy needed to move 1 Coulomb of electrons through a potential of 1 volt.

Note about 0.239 grams of water by 1 °C.

$F = \frac{9}{5}C + 32$

$$1\,\text{BTU} = 1\,\cancel{\text{lb}}\cdot\cancel{°\text{F}}\;\frac{5\;°\text{C}}{9\;\cancel{°\text{F}}}\;\frac{453.6\,\text{g}}{1\,\cancel{\text{lb}}} = 252\ \text{Calories}$$

$$252\ \cancel{\text{Calories}}\cdot\frac{1\ \text{Joule}}{0.239\ \cancel{\text{Calories}}} = 1054.4\ \text{Joules}$$



Definition: Heat Capacity ...

C = amount of energy per unit of mass per degree temp. $\sim \dfrac{[E]}{[M][u]}$

The heat capacity of water ...

$$C = 1\,\frac{\text{Calories}}{\text{grams}\cdot°\text{C}} \approx 1\,\frac{\text{BTU}}{\text{lb}\cdot°\text{F}} = 1\,\frac{\cancel{\text{Calories}}}{\text{grams}\cdot°\text{C}}\;\frac{1\ \text{J}}{0.239\ \cancel{\text{Cal}}}$$

$$= 4.1841\,\frac{\text{J}}{\text{g}\cdot°\text{C}} = 4.1841\,\frac{\text{J}}{\text{g}\cdot\text{K}}$$

Water at 25 °C	liquid	**4.1816**
Copper	solid	0.385
Aluminium	solid	0.897

Convert temperatures to energies..

Heat flux: φ

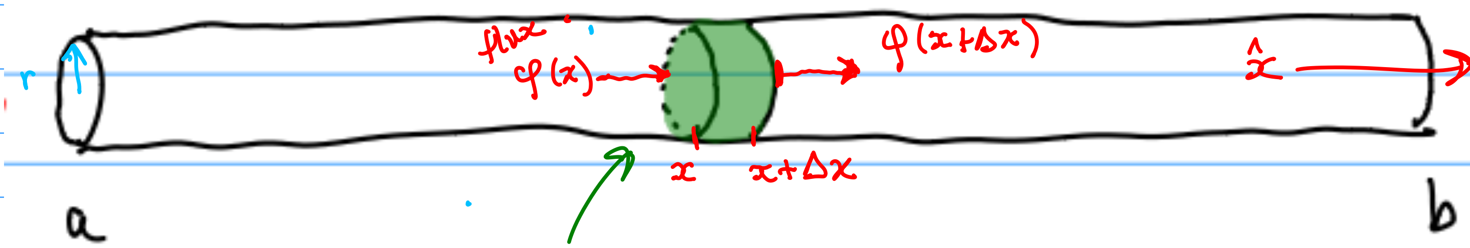
$$[\varphi] = \frac{[E]}{[L]^2[T]}$$

Heat source: Q

$$[Q] = \frac{[E]}{[L]^3[T]}$$

rate of change of heat energy in time	=	$\underbrace{\text{heat energy flowing across boundaries per unit time}}_{\varphi(x) - \varphi(x+\Delta x)}$	+	heat energy generated inside per unit time.

is is called **conservation of heat energy**. For the small slice, the rate of change of



how much energy is in the region between x and $x+\Delta x$

$$\boxed{E(t)} = \int_x^{x+\Delta x} \int_0^r \int_0^{2\pi} e(x,\cancel{r},\cancel{\theta},t)\, r\, dr\, d\theta \; dx$$

constant in r and θ

$$= \int_x^{x+\Delta x} \underbrace{(\pi r^2)}_{A} e(x,t)\,dx = A\int_x^{x+\Delta x} e(x,t)\,dx$$

$$[e(x,t)] = \frac{[E]}{[L]^3} = \frac{[E]}{[M][u]} \frac{[M]}{[L]^3} [u]$$

normal density ρ

temperature

$$\boxed{e(x,t)} = c(x)\,\rho(x)\,u(x,t)$$

energy density...

plug in later.

$$\frac{\partial}{\partial t}\overbrace{A\int_{x}^{x+\Delta x} e(x,t)\,dx}^{E(t)} \simeq A\big(\phi(x)-\phi(x+\Delta x)\big) + A\int_{x}^{x+\Delta x} Q(x,t)\,dx$$

$$\frac{\frac{\partial}{\partial t} A\int_{x}^{x+\Delta x} e(x,t)\,dx}{\Delta x} \simeq A\left(\frac{\phi(x)-\phi(x+\Delta x)}{\Delta x}\right) + \frac{A\int_{x}^{x+\Delta x} Q(x,t)\,dx}{\Delta x}$$

Now take $\Delta x \to 0$ to get a differential equation...