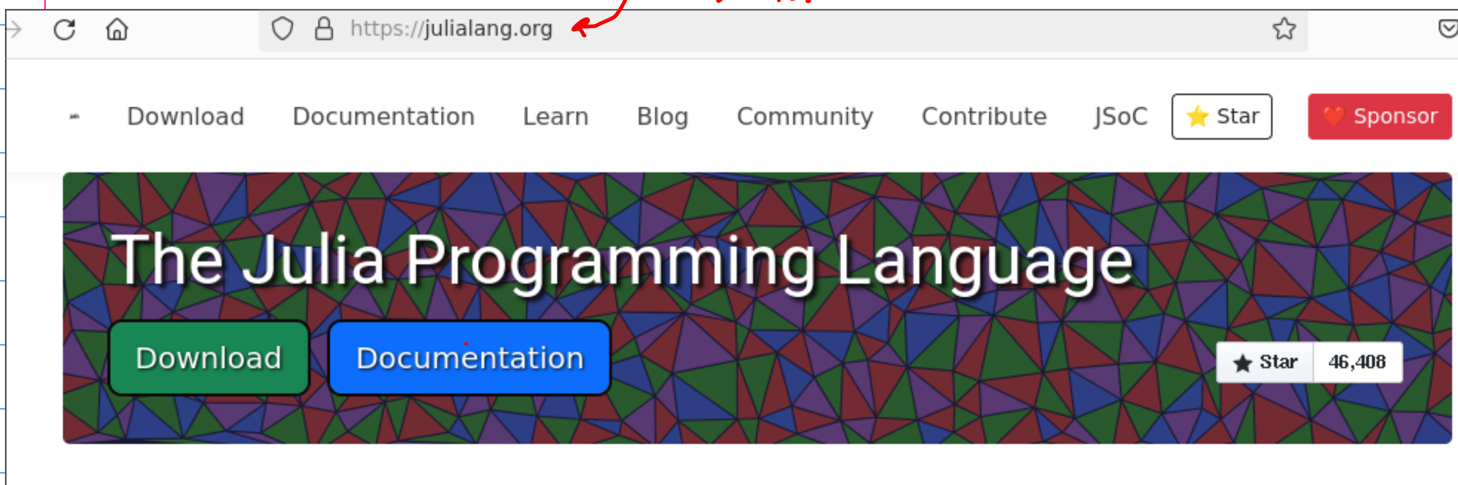
Use Julia to visualize the solution..

website...



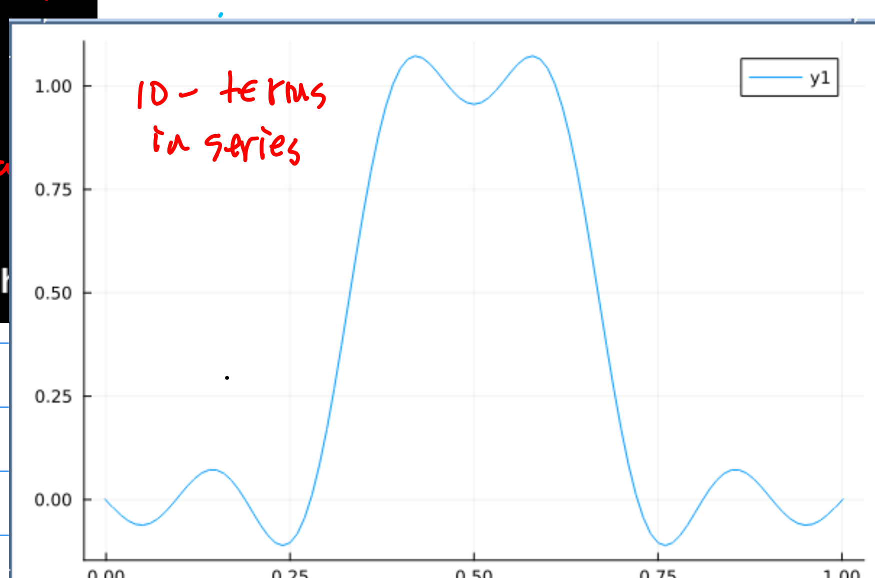

```
$ julia
               _
   _       _ _(_)_     |  Documentation: https://docs.julialang.org
  (_)     | (_) (_)    |
   _ _   _| |_  __ _   |  Type "?" for help, "]?" for Pkg help.
  | | | | | | |/ _` |  |
  | | |_| | | | (_| |  |  Version 1.6.7 (2022-07-19)
 _/ |\__'_|_|_|\__'_|  |  Official https://julialang.org/ release
|__/                   |

julia> u(x,t)=sum([-2/(n*pi)*(cos(2*n*pi/3)-cos(n*pi/3))*
                     sin(n*pi*x/L)*exp(-(n*pi/L)^2*k*t) for n=1:10])
u (generic function with 1 method)

julia> L=1; k=0.1;
```

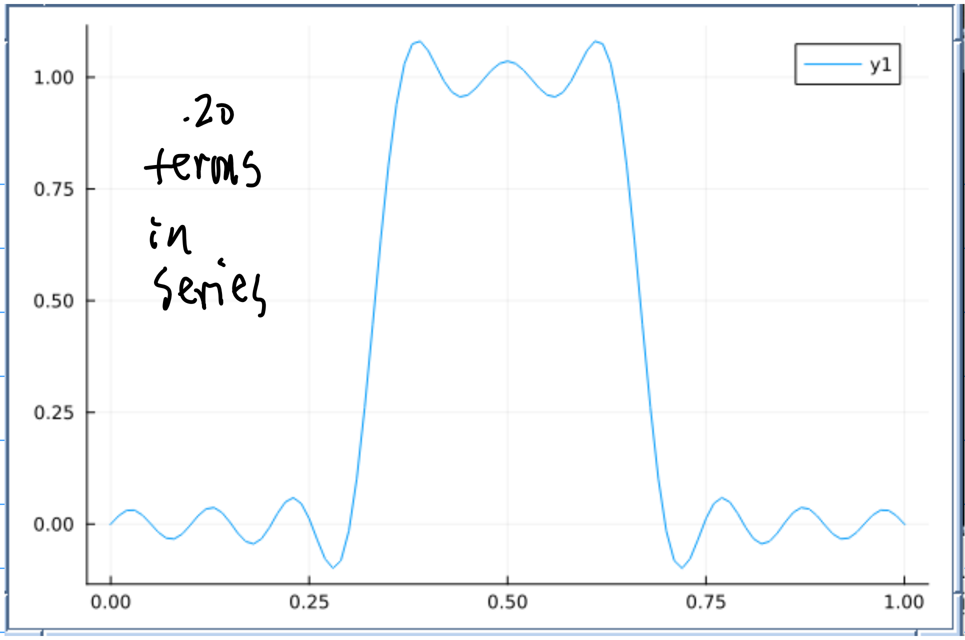
supposed to be inf. sum...

```
julia> using Plots

julia> X=0:0.01:1
0.0:0.01:1.0

julia> T=0:0.01:1
0.0:0.01:1.0

julia> f(x)=u(x,0)
f (generic function with 1 meth

julia> plot(X,f.(X))
```

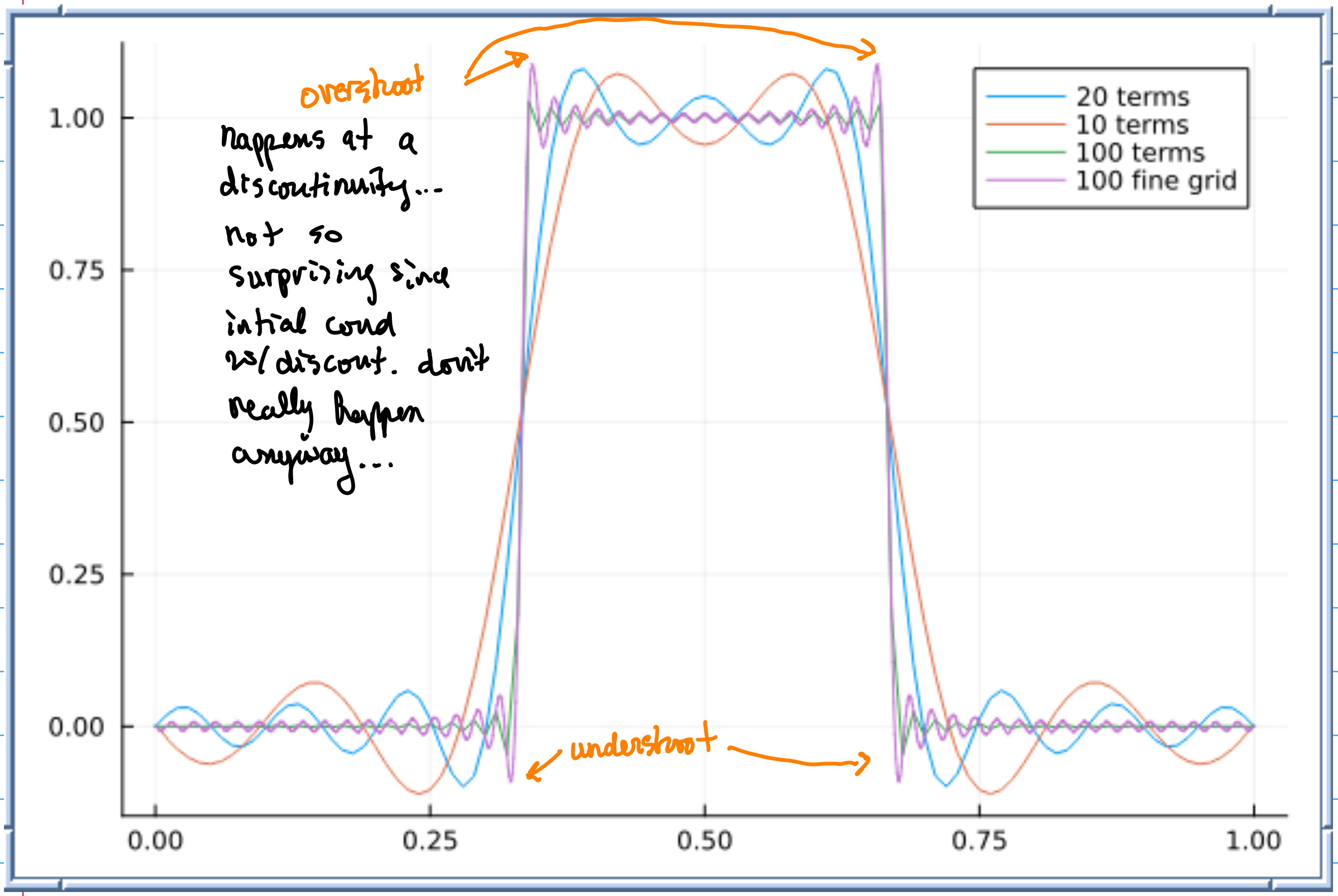
grids on [0,1]

Check initial condition

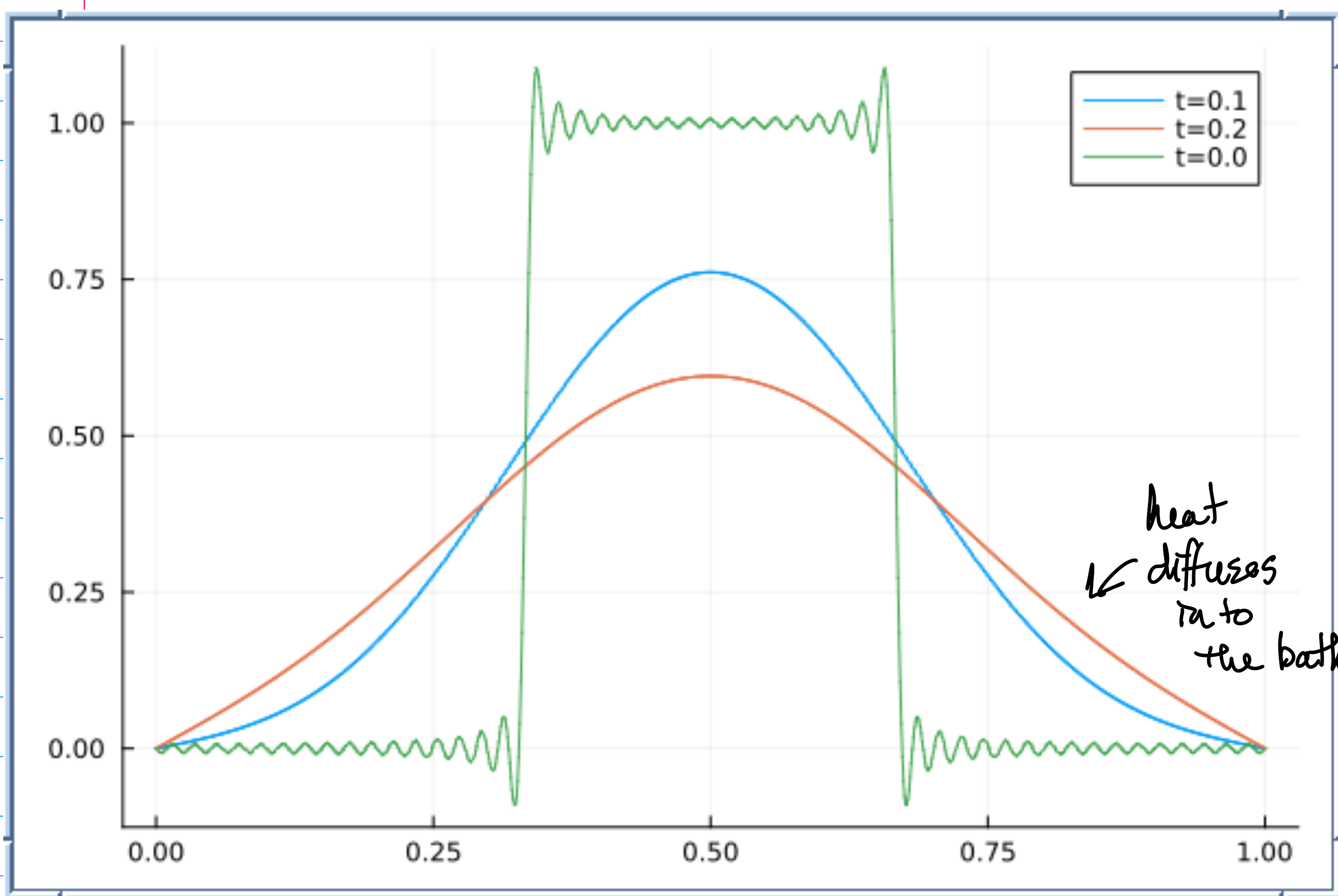





```
julia> u(x,t)=sum([-2/(n*pi)*(cos(2*n*pi/3)-cos(n*pi/3))*
                  sin(n*pi*x/L)*exp(-(n*pi/L)^2*k*t) for n=1:100])
u (generic function with 1 method)

julia> plot!(X,f.(X),label="100 terms")
```

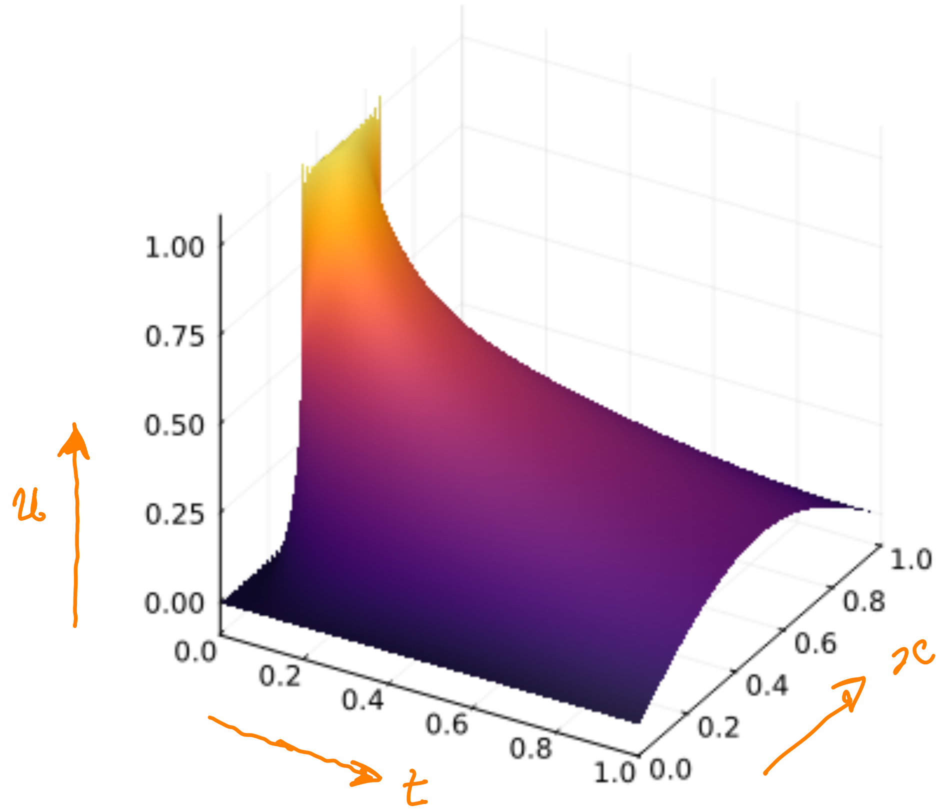


```
julia> g(x)=u(x,0.1)
g (generic function with 1 method)

julia> plot(X,g.(X),label="100 fine grid")

julia> g2(x)=u(x,0.2)
g2 (generic function with 1 method)

julia> plot(X,g.(X),label="t=0.1")

julia> plot!(X,g2.(X),label="t=0.2")

julia> plot!(X,f.(X),label="t=0.0")
```



heat diffuses into the bath

```
julia> surface(X,T,[u(x,t) for x in X,t in T])
```



Insulating boundary ...

The solution of the PDE

homogeneous

PDE: $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$ $\quad 0 < x < L$, $t > 0$

homogeneous

BC: $\frac{\partial}{\partial x}u(0,t) = 0$
$\frac{\partial}{\partial x}u(L,t) = 0$

Will solve this first

not homogeneous ..

IC: $u(x,0) = f(x)$.

$u(x,t) = \varphi(x)\, G(t)$

$\boxed{\text{Two ODEs} \qquad G'(t) = -\lambda k G(t)}$ and $\varphi''(x) = -\lambda \varphi(x)$

I.C. $u(x,0) = f(x)$ use superposition to solve this $\quad \varphi'(0) = 0 \quad \varphi'(L) = 0$

solve ODE — Solve $\varphi'' = -\lambda\varphi$ subject to $\varphi'(0) = 0 \quad \varphi'(L) = 0$

Case $\lambda = 0$: $\quad \varphi'' = 0$

Gen. soln $\quad \varphi(x) = C_1 x + C_2$

$\varphi'(x) = C_1$

$\varphi'(0) = C_1 = 0 \qquad$ so $\quad C_1 = 0$

$\varphi'(L) = 0 \qquad$ no new information

Thus $\quad \varphi(x) = C_2 \quad \leftarrow$ This is good because φ is not the zero function

Case $\lambda < 0$. $\quad \varphi'' = |\lambda|\varphi$

Gen soln $\quad \varphi(x) = c_1 e^{\sqrt{|\lambda|}\,x} + c_2 e^{-\sqrt{|\lambda|}\,x}$

$\varphi'(x) = c_1\sqrt{|\lambda|}\, e^{\sqrt{|\lambda|}\,x} - c_2\sqrt{|\lambda|}\, e^{-\sqrt{|\lambda|}\,x}$

$\varphi'(0) = c_1\sqrt{|\lambda|} - c_2\sqrt{|\lambda|} = 0 \qquad c_1 = c_2$

$\varphi'(L) = 2\left(\dfrac{c_1\sqrt{|\lambda|}\, e^{\sqrt{|\lambda|}\,L} - c_1\sqrt{|\lambda|}\, e^{-\sqrt{|\lambda|}\,L}}{2}\right)$

$= c_1\sqrt{|\lambda|}\,\sinh\left(e^{\sqrt{|\lambda|}\,L}\right) = 0 \qquad c_1 = 0$

$\varphi(x) = 0$