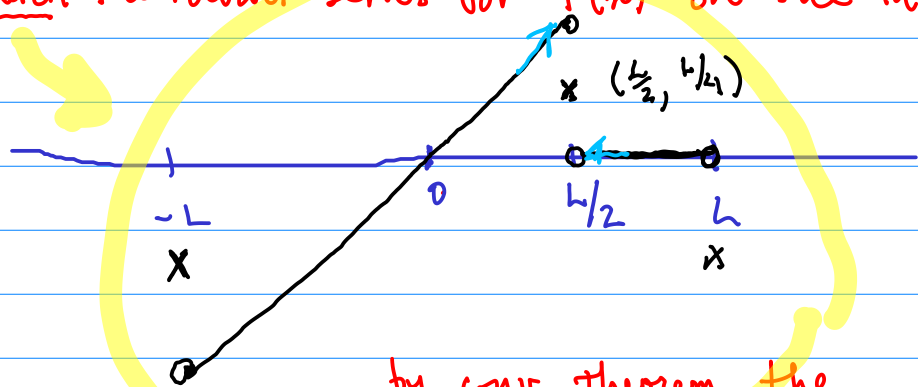
Take home Quiz 6 due on Friday. Please download the pdf file from the website. There is no need to print it, just answer the questions on a blank piece of paper.

Example

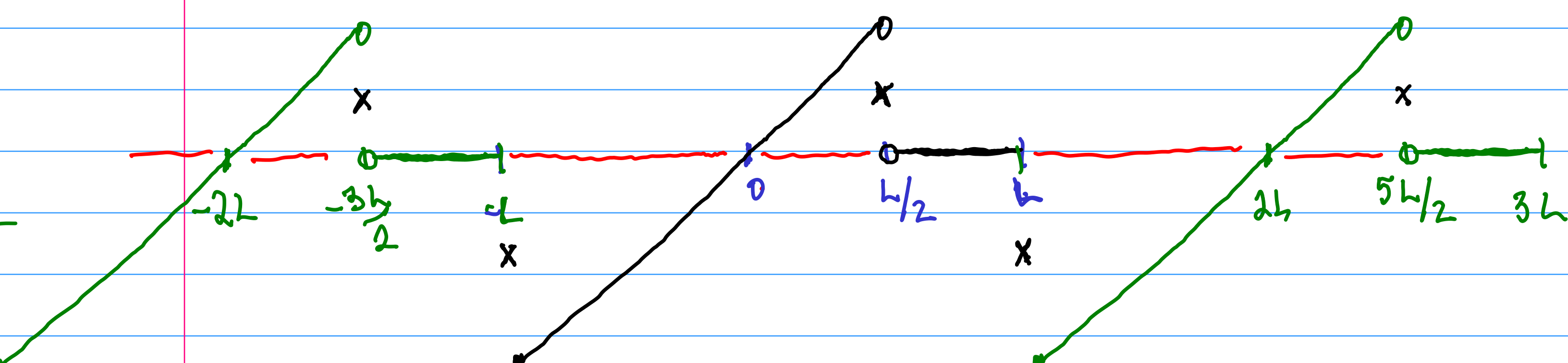
$$\text{(g) } f(x) = \begin{cases} x & x < L/2 \\ 0 & x > L/2 \end{cases}$$

Sketch the Fourier series for f(x) on the interval [-L, L].



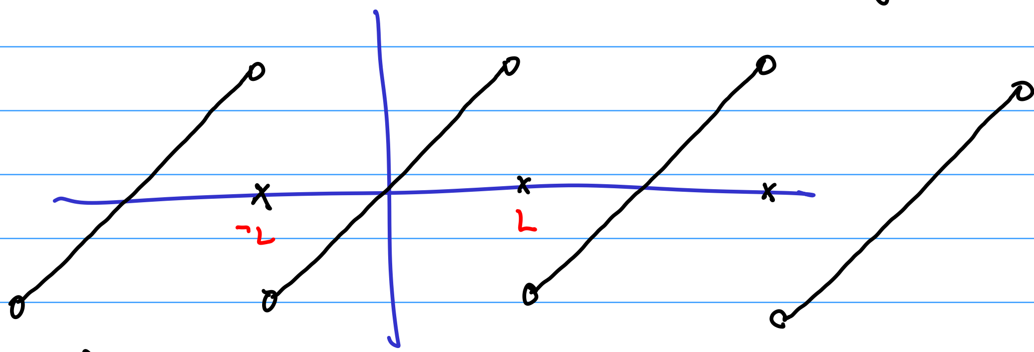
note the Fourier series is defined in terms of integrals and it doesn't matter how the endpoints of the pieces are defined

by conv. theorem, the series converges to f where f is smooth and the average at the jumps.



$f(x) = x$ on $[-L, L]$

Fourier series of f converges to this graph...



Actually find the series

$$a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}.$$

where

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

and

$$\begin{cases} a_n = \dfrac{1}{L}\displaystyle\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \\ b_n = \dfrac{1}{L}\displaystyle\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx. \end{cases}$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} x\,dx = 0$$

by parts

$$a_n = \frac{1}{L}\int_{-L}^{L} x\cos\frac{n\pi x}{L}\,dx = \frac{1}{L}\left(uv\Big|_{-L}^{L} - \int_{-L}^{L} v\,du\right)$$

$u = x$ $\qquad du = dx$

$dv = \cos\frac{n\pi x}{L}\,dx \qquad v = \frac{L}{n\pi}\sin\frac{n\pi x}{L}$

$$= \frac{1}{L}\left\{x\frac{L}{n\pi}\sin\frac{n\pi x}{L}\Big|_{-L}^{L} - \int_{-L}^{L}\frac{L}{n\pi}\sin\frac{n\pi x}{L}\,dx\right\} = L\left(\frac{L}{n\pi}\right)^2\cos\frac{n\pi x}{L}\Big|_{-L}^{L}$$

$$= L\left(\frac{L}{n\pi}\right)^2\left(\cos n\pi - \cos n\pi\right) = 0$$

Actually knew this before doing the calculation since $x\cos\frac{n\pi x}{L}$ is odd.

by parts

$$b_n = \frac{1}{L}\int_{-L}^{L} x \sin\frac{n\pi x}{L}\,dx = \frac{1}{L}\left(uv\Big|_{-L}^{L} - \int_{-L}^{L} v\,du\right)$$

$u = x \qquad du = dx$

$dv = \sin\frac{n\pi x}{L}\,dx \qquad v = \frac{-L}{n\pi}\cos\frac{n\pi x}{L}$

$$= \frac{1}{L}\left\{ x\,\frac{-L}{n\pi}\cos\frac{n\pi x}{L}\Big|_{-L}^{L} + \int_{-L}^{L}\frac{L}{n\pi}\cos\frac{n\pi x}{L}\,dx \right\} =$$

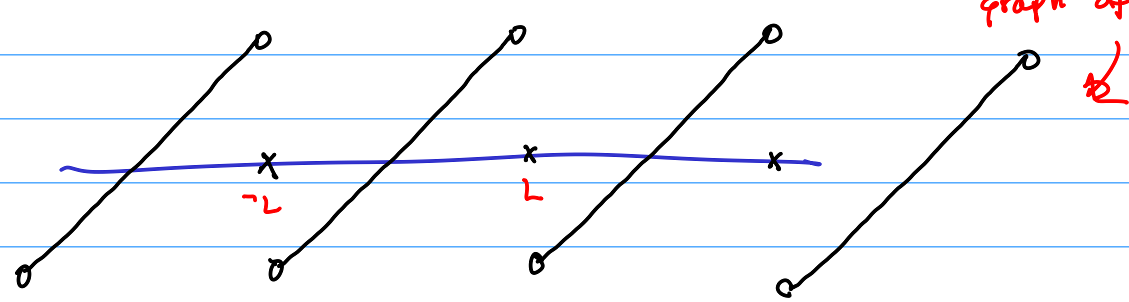
$$= \frac{1}{\cancel{L}}\left\{ 2\cancel{L}\,\frac{-L}{n\pi}\cos n\pi + \cancel{\left(\frac{L}{n\pi}\right)^2 \sin\frac{n\pi x}{L}\Big|_{-L}^{L}} \right\}$$

$$= \frac{-2L}{n\pi}\cos n\pi = \frac{-2L}{n\pi}(-1)^n$$

$\cos 0 = 1 \qquad \cos\pi = -1 \qquad \cos 2\pi = 1 \qquad \cos 3\pi = -1$

The Fourier series is

$$\cancel{a_0} + \sum_{n=1}^{\infty} \cancel{a_n} \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}. = \underbrace{\sum_{n=1}^{\infty} \frac{-2L}{n\pi}(-1)^n \sin\frac{n\pi x}{L}}_{\text{graph of this}}$$

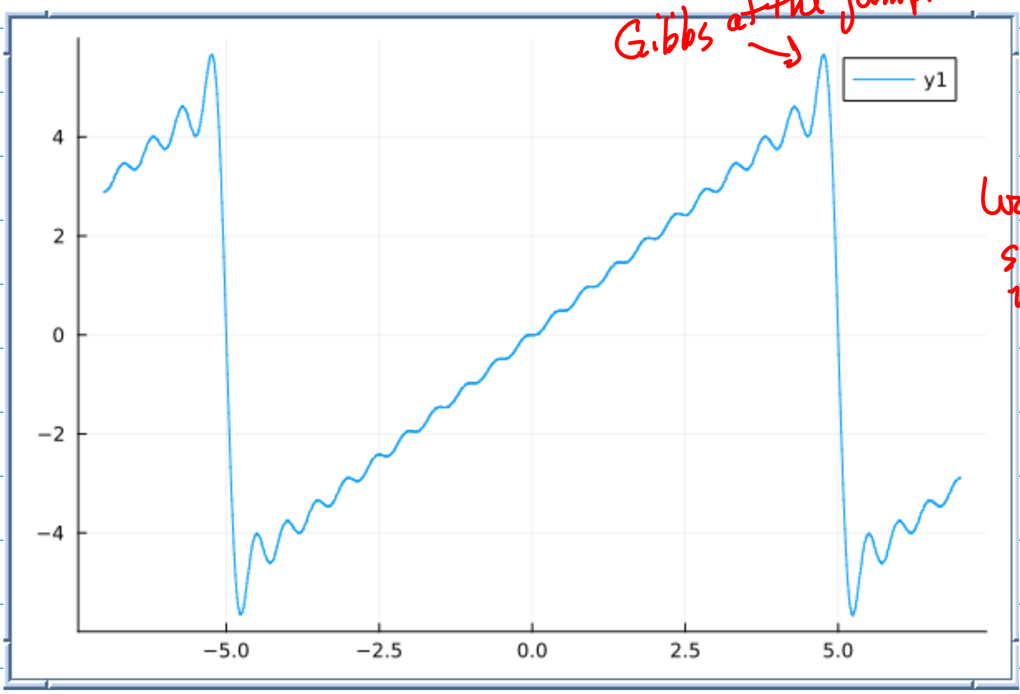


Check this...

Verify numerically...

```
julia> s(x)=sum([-2*L/(n*pi)*(-1)^n*sin(n*pi*x/L) for n=1:20])
s (generic function with 1 method)

julia> using Plots

julia> L=5
5

julia> xs=-7:0.01:7
-7.0:0.01:7.0

julia> plot(xs,s.(xs))
```

Gibbs at the jump.



Look like a sawtooth as it was supposed to

but in the limit it converges to something with a jump discontinuity and no oscillations.

Recall... we want to use the Fourier series to represent

- Initial distribution of heat
- Solution to a PDE.

$$g(x) = \sum_{n=1}^{\infty} \frac{-2L}{n\pi}(-1)^n \sin\frac{n\pi x}{L}$$

what is $g'(x)$?

$$g'(x) = \frac{d}{dx}\sum_{n=1}^{\infty} \frac{-2L}{n\pi}(-1)^n \sin\frac{n\pi x}{L}$$

interchange of limiting process is dangerous we need some math to do this..

$$= \sum_{n=1}^{\infty} \frac{-2L}{n\pi}(-1)^n \frac{d}{dx}\sin\frac{n\pi x}{L}$$

$$= \sum_{n=1}^{\infty} \frac{-2\cancel{L}}{\cancel{n\pi}}(-1)^n \frac{\cancel{n\pi}}{\cancel{L}}\cos\frac{n\pi x}{L}$$

$$= \sum_{n=1}^{\infty} \underbrace{-2(-1)^n \cos\frac{n\pi x}{L}}$$

general term in this series doesn't tend to 0

this series doesn't converge